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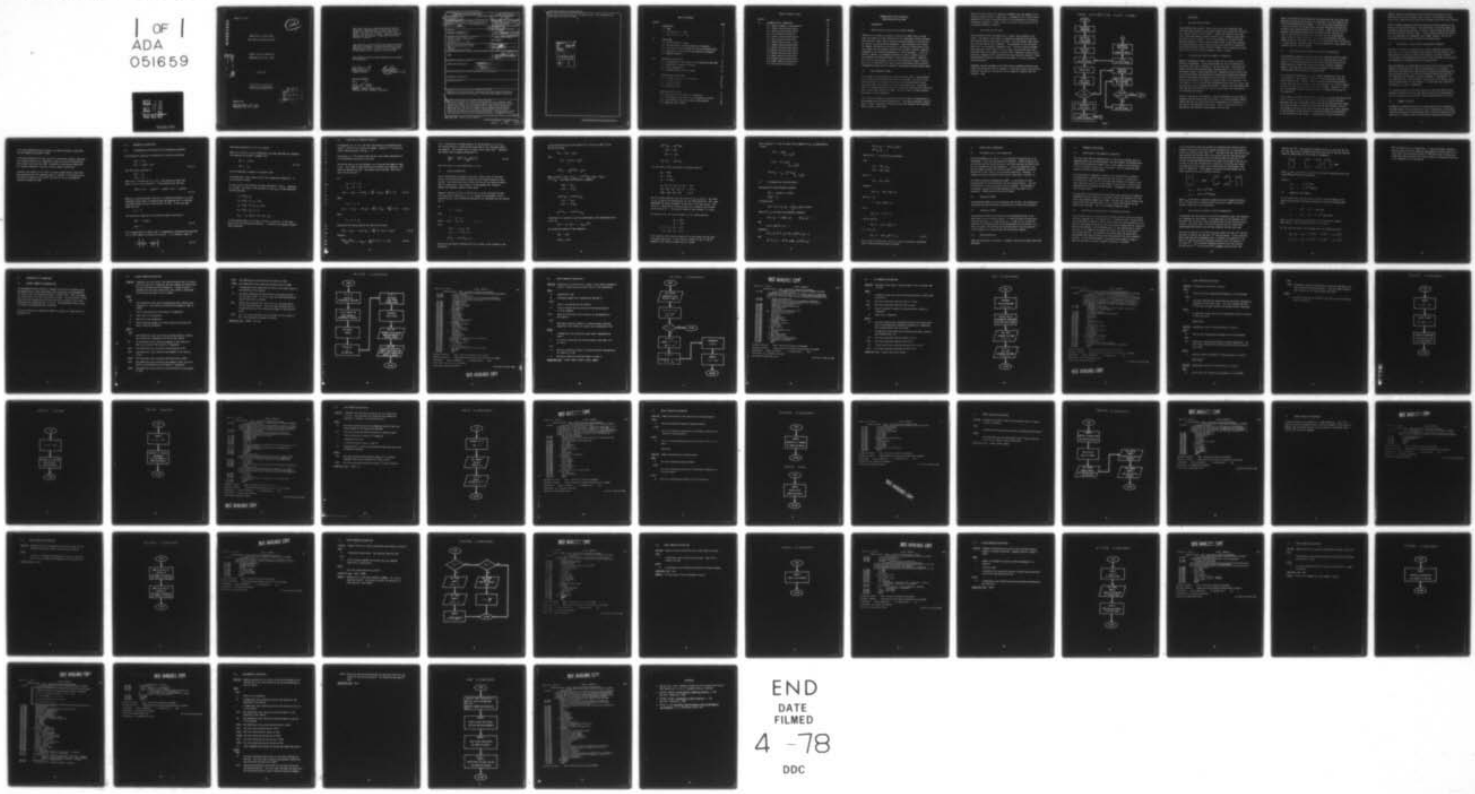

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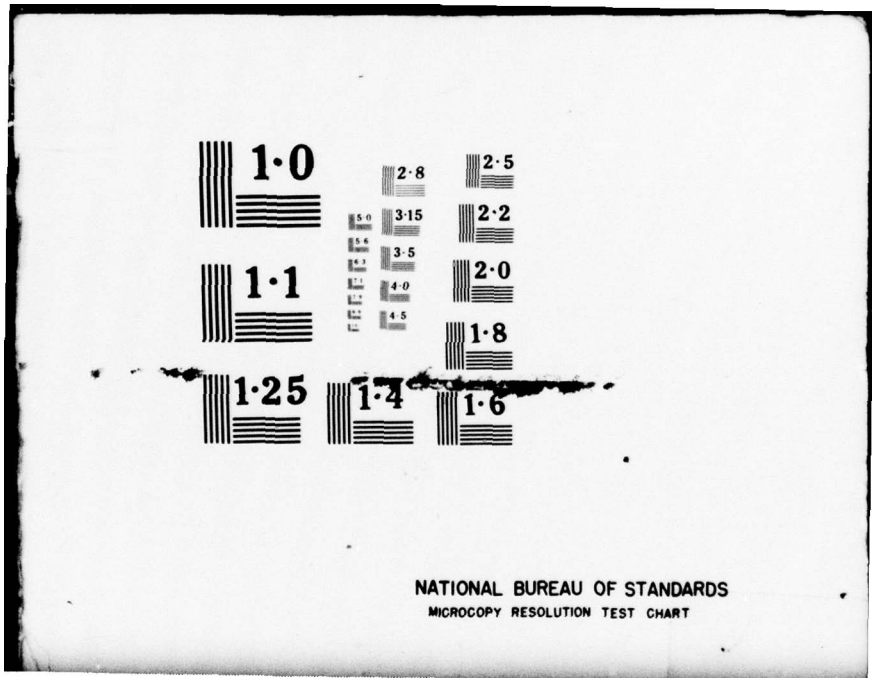
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SAMTEC TR 77-223

COMPUTATION OF STATE VECTOR AND TRANSITION MATRIX FOR TRAM

FEDERAL ELECTRIC CORPORATION
VANDENBERG AFB, CALIF. 93437

1 JUNE 1977

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Vandenberg AFB, Calif. 93437

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This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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This report contains the analysis and documentation of subroutines which contain the capability to efficiently compute the state vector and transition matrix needed for the Kalman Filter in TRAM, a post flight estimation program. The analysis contains the development and testing of algorithms to numerically integrate, perform numerical partial differentiation and interpolate using Spline polynomials. The subroutines, which are documented in this report, use these algorithms to compute the state

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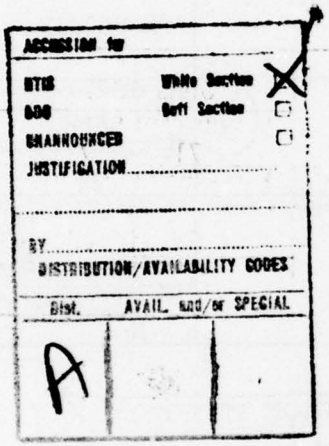
vector and transition matric for the Kalman Filter. This capability has more general use than for TRAM.



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COMPUTATION OF STATE VECTOR AND TRANSITION MATRIX FOR TRAM

I. INTRODUCTION

1.1 TRAM (Trajectory Reconstruction Analysis Method)

TRAM will be a system of subroutines produced for SAMTEC which, running under the control of a main program, will use telemetered inertial guidance data and metric sensor data to estimate selected parameters of the missile, the sensors, the earth and the atmosphere. These parameters will include, for example, the position and velocity of the vehicle, coefficients of the error models for the guidance system and for the metric sensors. Normally, parameters of the earth and atmosphere will not be estimated. It will be possible, however, to propagate the effect of errors of these, or any of the parameters not being estimated, on those being estimated. TRAM will be useful for many purposes, such as mission planning and special studies, but it will be used mainly for metric sensor performance analysis and will subsequently become a part of the Metric Integrated Processing System (MIPS).

1.2 The Estimation in TRAM

The parameters being estimated are called the state vector. The estimation is an iterative process which takes place in two steps. The first is the optimal estimation of the state vector and its covariance matrix at any given time using all the data up to and including that time. The second step is to compute the optimal estimate of the state vector and its covariance matrix at any time using all the data. The first step is called filtering. The second is called smoothing.

The filtering in TRAM will be done by the Carlson square root adaptation of the extended Kalman filtering algorithm [1]. The Kalman algorithm is applicable to linear systems and in this application the equations are linearized about a nominal state vector.

There will be two options for smoothing, dependent upon the segment of the trajectory where the data is taken from. In powered flight a fixed interval smoother will be used, and during free fall, the smoothing will be done by retrograde integration. Figure 1 gives a functional flow of the filtering and smoothing TRAM will perform.

1.3 The Purpose of this Study

The calculations within the dashed box in Figure 1 were studied in this effort. The nominal state vector is computed by integrating the position and velocity from one data time to the next. With the nominal state vector, the transition matrix can be computed. The computation of the transition matrix will be by numerical integration also. Since the intervals between data times are normally small (.1 sec or less), and the state vector and transition matrix are needed at each data point, computation times would be excessive. Because of this, more efficient algorithms were required. These were devised and numerical experiments were made to test their execution time and accuracy.

Therefore, the main purpose of this effort was to establish efficient algorithms and subroutines that will compute (1) the state vectors and transition matrices, (2) numerical derivatives and (3) numerical integrals that are needed for TRAM.

TRAM ESTIMATION FLOW CHART

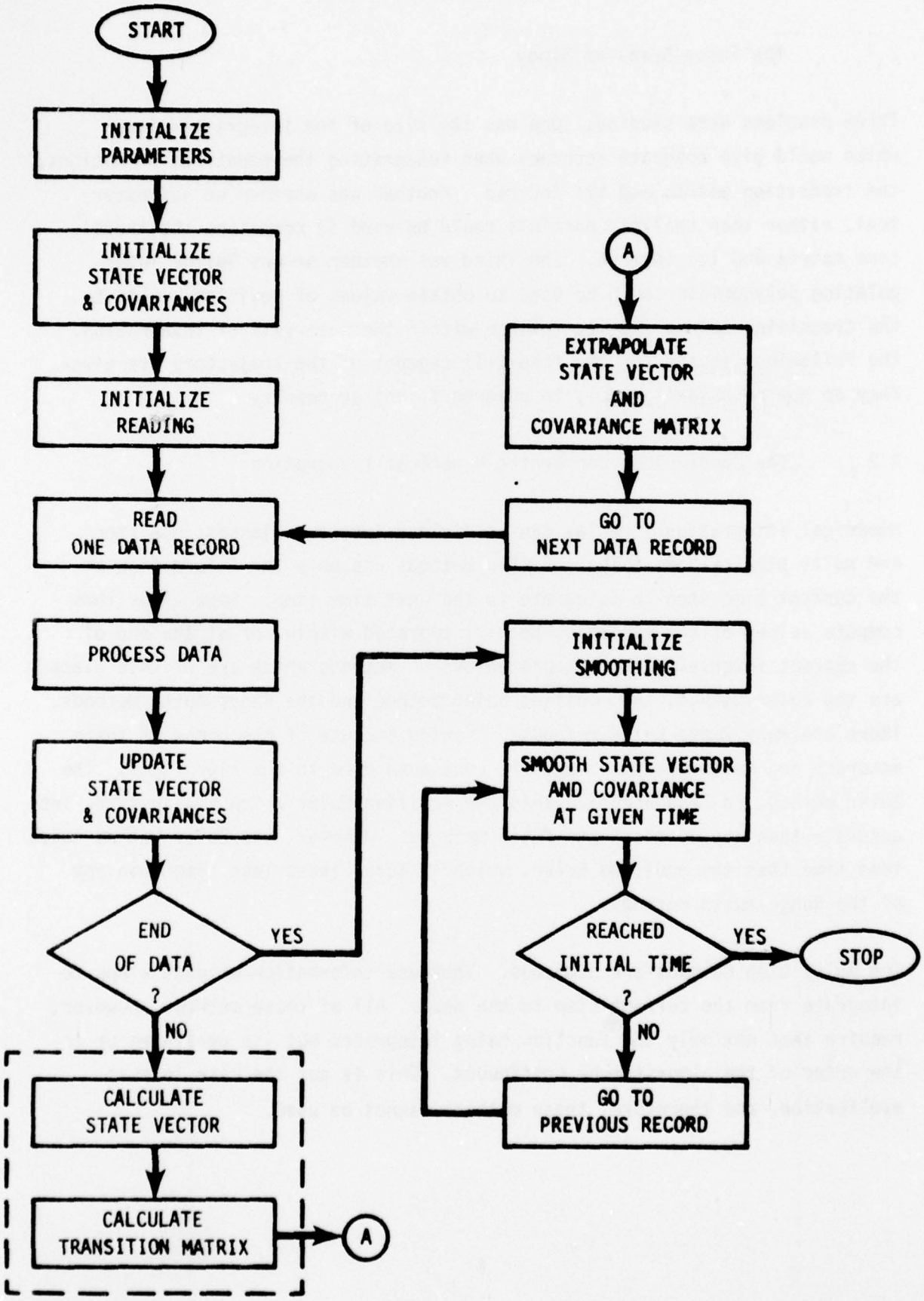


FIGURE 1

II CONCLUSIONS

2.1 The Three Areas of Study

Three problems were studied. One was the size of the integration step which would give adequate accuracy when integrating the equations of motions, the transition matrix and its inverse. Another was whether or not numerical, rather than analytic partials could be used in computing the transition matrix and its inverse. The third was whether or not Spline interpolating polynomials could be used to obtain values of position, velocity, the transition matrix and its inverse within the intervals of integration. The following results for the free-fall segment of the trajectory are given. They do not necessarily apply to powered flight or reentry.

2.2 The Conclusions Concerning Numerical Integration

Numerical integration formulas can be divided into two classes, one step and multi step methods. The one step methods use only the information at the current time step to integrate to the next time step. Some algorithms compute values of the variables being integrated within, or at the end of the current interval being integrated over. Methods which are of this class are the Euler method, the modified Euler method and the Runge-Kutta methods. There are many Runge-Kutta methods differing because of the order of their accuracy and because of the specific constants used in the algorithms. The Euler method, is less accurate than the modified Euler which is, in turn, less accurate than any of the Runge-Kutta methods. However, the Euler method takes less time than the modified Euler, which in turn, takes less time than any of the Runge-Kutta methods.

The multi step methods are numerous. They use information at past steps to integrate from the current step to the next. All of these methods, however, require that not only the function being integrated but its derivates up to the order of the algorithm be continuous. This is not the case in this application, and therefore, these methods cannot be used.

Results from numerical experimentation have determined that the fourth order Runge-Kutta gives the best accuracy and computation time when compared with the Euler and modified Euler methods. It has further been determined that a step size of five (5) seconds may be used when integrating the equations of motion, the transition matrix and its inverse. These results are valid because, even though the Runge-Kutta takes more time per step than the Euler or modified Euler methods take, the step size can be appreciably larger because of the much greater accuracy of the Runge-Kutta algorithm. Another reason for the larger step size is that during free-fall, acceleration is slowly varying. During powered flight the acceleration is much greater, and a smaller step size will be needed.

2.3 Conclusions Concerning Numerical Partial Differentation

The differential equations of motion need to be differentiated with respect to position and velocity in order to obtain the transition matrix and its inverse. This study has shown that these partial derivatives can be computed numerically, rather than analytically, with a central difference formula. The study was made by comparing the results of the numerical differentiation with the analytic differentiation.

If the analytic differentiation is done without mathematical errors, and the evaluation of the formulas is done with infinite precision, then the values would be exact. However, analytic partial differentiation normally becomes very detailed and numerous terms are developed. As a result, chances of error are high and very thorough checking is needed. Moreover, if there are a large number of terms, round off errors can degrade the accuracy of the computation.

Often, numerical partial differentiation is simple to program compared with the analytic approach. Once a subroutine to evaluate the function has been developed, all that is required to obtain the partial derivative with respect to a given variable is to evalute the function for two different values of the variable, leaving all the other variables the same; then, form a quotient of the difference of the values of the function divided by the difference of the two values of the variable. Although this is easy to perform on a

computer, there are truncation and round off errors associated with this method which the analytic approach does not have. The reduction of these errors to an insignificant value was one of the goals which this study achieved.

There is another advantage to obtaining partial derivatives numerically and that is it is much more flexible. The acceleration calculations depend upon the model of gravity acceleration, of course. There are many gravity models and even the form of them can change. To write down the partial derivatives of all of them would be a monumental task indeed. However, partials can be obtained relatively simply by numerical differentiation.

2.4 Conclusions on Using Spline Interpolating Polynomials

The position, velocity, transition matrix can be integrated accurately at step sizes up to five (5) seconds during free-fall, but the values of these quantities will be needed at every tenth second or at smaller intervals. In this study it was determined that values at the finer mesh can be obtained satisfactorily by interpolation with Spline polynomials.

Spline polynomials are constructed such that the function and a specified number of its derivatives fit data at the end points of an interval. In the case of the state vector, the position, velocity, and acceleration are given at the end points. For each of the three variables this specifies six conditions, and therefore, a quintic polynomial is determined. For the transition matrix and its inverse, the function and its derivative are given at the end points of the interval. This specifies four conditions, and therefore, a cubic is determined.

This study has shown also that the velocities in the state vector can be obtained satisfactorily at the finer mesh by evaluating the derivatives of the Spline interpolating polynomials that have been derived for position.

2.5 Summary of Results

The amount of computation time can be reduced appreciably by integrating the state vector, the transition matrix and its inverse over a five second interval and then obtaining these quantities at the finer intervals of the data rate (0.1 second or less) by using Spline interpolating polynomials.

It has been determined that the accuracy of these algorithms is sufficient for the estimation performed in TRAM.

It has been determined also that the partial derivatives needed to calculate the transition matrix can be computed by numerical partial differentiation with accuracy sufficient for TRAM. Computation of derivatives numerically will improve computation times and allow for greater flexibility.

Therefore, the purpose of this effort has been accomplished and algorithms and subroutines have been developed that will compute, (1) the state vectors and transition matrices, (2) partial derivatives, and (3) the numerical integration needed for TRAM.

III MATHEMATICAL DESCRIPTION

3.1 The Mathematical Description of the Integration Algorithm

The differential equations of the motion of a vehicle are given by

$$\begin{aligned} \dot{p}(t) &= v(t) \\ \dot{v}(t) &= a(p(t), v(t)) \end{aligned} \tag{3.1.1}$$

with the initial condition of

$$\begin{aligned} p(o) &= p_o \\ v(o) &= v_o \end{aligned}$$

where $p(t)$ is the position at t, $v(t)$ is the velocity at time t and $a(p(t), v(t))$ is the acceleration. The acceleration has the form

$$a(p(t), v(t)) = a_g(p(t)) + a_r(p(t), v(t)) + a_d(v(t)) \tag{3.1.2}$$

Where a_g is the gravitational term and is a function of position, a_r is the acceleration which would be sensed because the equations are in a rotating coordinate system, and a_d is the drag and lift term. In the case under study free fall,

$$a_d = 0.$$

The differential equations of the transition matrix are given by

$$\begin{aligned} \dot{Q}(t) &= F(t)Q(t) \\ Q(0) &= I \end{aligned} \tag{3.1.3}$$

Q is a square matrix of order 6 and F is determined by differentiating equations (3.1.1) with respect to position and velocity. F is, therefore, equal to

$$F = \begin{pmatrix} 0 & I \\ \frac{\partial a(p,v)}{\partial p^T} & \frac{\partial a(p,v)}{\partial v^T} \end{pmatrix} \tag{3.1.4}$$

where each partition of F is a 3 x 3 matrix.

Both (3.1.1) and (3.1.3) are integrated by the same algorithm and, therefore, the equations are written in general form

$$\dot{X}(t) = f(X,t)$$
$$X(o) = X_o \qquad (3.1.5)$$

and the algorithm is applied to the general form.

The method used is the standard fourth order Runge-Kutta method [2]. It is described as follows.

Let the values of the variables be given determined at time t_n. Therefore, X_n is known. The values of the variables at time t_{n+1} are then determined by these equations.

$$K_1 = hf(X_n, t_n)$$
$$K_2 = hf(X_n + K_1, t_n + h/2)$$
$$K_3 = hf(X_n + 1/2\ K_2, t_n + h/2)$$
$$K_4 = hf(X_n + K_3, t_n + h)$$
$$X_{n+1} = X_n + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

In the formulas above h is the step size and the function f is the right hand side of the differential equations. X represents the unknown variables being integrated.

3.2 Algorithms for Numerical Partials

From equation (3.1.4) it is seen that the partials of acceleration with respect to position and velocity are needed. Equation (3.1.2) gives the form of the acceleration, with $a_d = 0$.

The gravity a_r is the rotation term and has a very simple form which can be differentiated analytically with ease.

The gravity term a_g may vary markedly in its form and the number of terms it has. This leads to complicated analytic partials and, therefore, the numerical approach was used. The equation can be derived simply from a Taylor series expansion.

Let

$$X_o + \Delta X = X_1$$
$$X_o - \Delta X = X_{-1}$$

$$f(X_1) = f(X_o) + \Delta X f'(X_o) + \frac{\Delta X^2}{2!} f''(X_o) + \frac{\Delta X^3}{3!} f'''(\eta) \qquad (3.2.1)$$

where

$$X_o \leq \eta \leq X_1$$

$$f(X_{-1}) = f(X_o) - \Delta X f'(X_o) + \frac{\Delta X^2}{2!} f''(X_o) - \frac{\Delta X^3}{3!} f'''(\xi) \qquad (3.2.2)$$

where

$$X_{-1} \leq \xi \leq X_o$$

Subtracting the second equation from the first, we have

$$f(X_1) - f(X_{-1}) = 2\Delta X f'(X_o) + \frac{\Delta X^3}{3!} [f'''(\eta) + f'''(\xi)] \qquad (3.2.3)$$

Therefore

$$\frac{f(X_1) - f(X_{-1})}{2\Delta X} = f'(X_o) + \frac{\Delta X^2}{12} [f'''(\eta) + f'''(\xi)] \qquad (3.2.4)$$

This is the central difference formula for the derivation of $f(X)$ at X_o. The truncation error is given by the second term on the right hand side of the equation. The trucation error is seen to be of order $(\Delta X)^2$. Therefore, the formula used to compute the partials is

$$\frac{\partial A_g(X)}{\partial X} = \frac{A_g(X + \Delta X) - A_g(X - \Delta X)}{2\Delta X} \qquad (3.2.4)$$

where the value of ΔX was determined to be five.

3.3 Spline Interpolation

Spline interpolating polynomials are used to obtain values of the state vector and the transition matrix within the five second interval of integration. The state vector uses a quintic Spline, and the transition matrix and its inverse uses a cubic Spline. The development that follows is general and therefore, applies to both cases.

Suppose a function $f(t)$ and its derivatives up to and including nth order are given over an interval $t_a \leq t \leq t_b$. Because of improvements in the round off errors, the interpolating polynomials are developed over the interval $[0, 1]$.

Let

$$u = (t - t_a)/T$$

where

$$T = t_b - t_a.$$

Define $g(u) \equiv f(t_a + uT) \qquad 0 \leq u \leq 1$

Then

$$\begin{aligned} g'(u) &= T\, f'(t_a + uT) \\ g''(u) &= T^2\, f''(t_a + uT) \\ &\vdots \\ g^{(n)}(u) &= T^n\, f^{(n)}(t_a + uT) \end{aligned}$$

where the prime denotes differentiation with respect to the variable in the parenthesis.

Let the spline for g(u) on the interval [0, 1] be $\tilde{g}(u)$; and $\tilde{f}(t)$ be defined as follows:

$$\tilde{f}(t) \equiv \tilde{g}((t - t_a)/T)$$

Then

$$\tilde{f}'(t) = \frac{1}{T}\tilde{g}'((t - t_a)/T)$$

$$\vdots$$

$$\tilde{f}^{(n)}(t) = \frac{1}{T^n}\tilde{g}^{(n)}((t - t_a)/T)$$

What is given is $f(t_a)$, $f'(t_a)$,..., $f^{(n-1)}(t_a)$, $f(t_b)$, $f'(t_b)$,..., $f^{(n-1)}(t_b)$. From these the following is computed.

$$g(0) = f(t_a)$$

$$g'(0) = Tf'(t_a)$$

$$\vdots$$

$$g^{(n-1)}(0) = T^{n-1}f^{(n-1)}(t_a)$$

$$g(1) = f(t_b)$$

$$g'(1) = Tf'(t_b)$$

$$\vdots$$

$$g^{(n-1)}(1) = T^{n-1}f^{(n-1)}(t_b)$$

From these it is possible to get the coefficients of the normalized Spline on [0, 1]:

$$\tilde{g}(u) = A_0 + A_1u + \ldots + A_{2n-1}u^{2n-1},$$

by solving the system of linear equations

$$\tilde{g}(0) = g(0)$$

$$\tilde{g}'(0) = g'(0)$$

$$\vdots$$

$$\tilde{g}^{(n-1)}(0) = g^{(n-1)}(0)$$

$$\tilde{g}(1) = g(1)$$

$$\tilde{g}'(1) = g'(1)$$

$$\vdots$$

$$\tilde{g}^{(n-1)}(1) = g^{(n-1)}(1)$$

For the quintic Spline the system of equations would be

$$A_0 = g(0)$$

$$A_1 = g'(0)$$

$$A_2 = g''(0)/2$$

$$A_5 + A_4 + A_3 + A_2 + A_1 + A_0 = g(1)$$

$$5A_5 + 4A_4 + 3A_3 + 2A_2 + A_1 = g'(1)$$

$$20A_5 + 12A_4 + 6A_3 + 2A_2 = g''(1)$$

The first three equations give the values for A_0, A_1, and A_2. The values for A_5, A_4 and A_3 are determined by the last three equations. This, then, determines the Spline polynomial for position in the state vector. The velocity is determined by differentiating this polynomial and dividing the derivative by T. This would be done for each variable, x, y and z.

The equations for the transition matrix or its inverse would be

$$A_0 = g(0)$$

$$A_1 = g'(0)$$

$$A_3 + A_2 + A_1 + A_0 = g(1)$$

$$3A_3 + 2A_2 + A_1 = g'(1)$$

These equations can be easily solved and the coefficients for the cubic are thereby determined. A cubic Spline is needed for each of the 36 elements of the transition matrix and its inverse.

Then in general, $\tilde{f}$ is the nth order Spline segment on $[t_a, t_b]$ approximating f, where

$$\tilde{f}(t) = \tilde{g}(u)\Big|_{u = \frac{t - t_a}{T}}$$

$$\tilde{f}'(t) = \frac{1}{T}\ \tilde{g}'(u)\Big|_{u = \frac{t - t_a}{T}}$$

$$\vdots$$

$$\tilde{f}^{(n-1)}(t) = \frac{1}{T^{n-1}}\ \tilde{g}^{(n-1)}(u)\Big|_{u = \frac{t - t_a}{T}}$$

3.4 The Generalized Transition Matrix

The solution of the differential equation

$$\dot{X}(t) = F(t)X(t) + G(t)u(t)$$
$$X(t_o) = X_o$$

is given by [3].

$$X(t) = \Phi(t, t_o)\left[X_o + \int_{t_o}^{t} \Phi(t_o, \sigma)G(\sigma)\, u(\sigma)d\sigma\right]$$

where $\Phi(t, t_o)$ satisfies the differential equation

$$\dot{\Phi}(t, t_o) = F(t)\Phi(t, t_o), \qquad \Phi(t_o, t_o) = I$$

Now

$$\Phi(t, t_o)\Phi^{-1}(t, t_o) = I$$

Therefore

$$\dot{\Phi}(t, t_o)\,\Phi^{-1}(t, t_o) + \Phi(t, t_o)\dot{\Phi}^{-1}(t, t_o) = 0$$

and

$$\dot{\Phi}^{-1}(t, t_o) = -\Phi^{-1}(t, t_o)\dot{\Phi}(t, t_o)\,\Phi^{-1}(t, t_o)$$

Therefore

$$\dot{\Phi}^{-1}(t, t_o) = -\Phi^{-1}(t, t_o)F(t)$$
$$\Phi^{-1}(t_o, t_o) = I$$

Now let $u(t) = 0$, and let X_o be arbitrary.

Then

$$X(t) = \Phi(t, t_o)X_o$$
$$X(s) = \Phi(s, t_o)X_o$$

But also

$$X(s) = \Phi(s, t)X(t)$$

Therefore

$$\Phi(s, t_o) = \Phi(s, t)\Phi(t, t_o)$$

Now let $s = t_o$

$$I = \Phi(t_o, t)\Phi(t, t_o)$$

and so

$$\Phi(t_o, t) = \Phi^{-1}(t, t_o)$$

From the equation

$$\Phi(s, t) = \Phi(s, t_o)\Phi(t_o, t)$$

It is clear that

$$\Phi(s, t) = \Phi(s, t_o)\Phi^{-1}(t, t_o) \qquad (3.4.1)$$

Since s and t are arbitrary, (3.4.1) is a way to calculate a transition matrix from one arbitrary point to another.

IV DESCRIPTION OF SUBROUTINES

4.1 The Capabilities of the Subroutines

With the mathematics set forth, it is now possible to describe how it was implemented. There were two subroutines developed. INTCON integrates the state vector and the transition matrix, and then calculates the coefficients for the interpolating Spline polynomials. It uses the Runge-Kutta integration scheme, and when the transition matrix is integrated, the partial derivatives in the F matrix, are calculated numerically. Moreover, it computes the Spline polynomial coefficients as described in Section 3.4.

The second subroutine developed was INTERP. It calculates the interpolated values for the state vector and the transition matrix. The transition matrix is from one arbitrary point to another. The mathematics of Sections 3.3 and 3.4 have been employed in the algorithms in this subroutine. INTERP must be called after INTCON.

4.2 Subroutine INTCON

The subroutine INTCON calls two subroutines INT and CON. INT performs the integration and CON calculates the coefficients of the Spline polynomials.

4.3 Subroutine INTERP

The subroutine INTERP first calculates the interpolated values of the state vector for an arbitrary time t_i. It then calculates the transition from the last time t_{i-1} to the current t_i. It does this by finding the inverse of the transition matrix from the beginning of the interval to t_{i-1}. Then it finds by interpolation the transition matrix from the beginning of the interval to t_i and multiples the two matrices together.

4.4 Other Subroutines

These two subroutines call others. Altogether there were fourteen subroutines developed.

V. NUMERICAL VERIFICATION

5.1 Verification of the Numerical Integration

The first steps taken to determine which of the one step methods would be adequate were to run the three methods, the Euler method, the modified Euler method and the Runge-Kutta method, over a large segment of the trajectory and see if adequate answers could be obtained. Only the Runge-Kutta gave answers which were accurate enough.

With the method of integration selected it was then possible to determine an optimum step size which would give the best accuracy. A step size of one second and a step size of five seconds gave answers which agreed to more than 0.001 foot in position and to 0.00001 foot per second in velocity when integrated over an interval of 1500 seconds. Consequently, the five second step size was chosen.

The magnitudes of the round off and transition errors caused by the numerical integration are well below errors from other sources and therefore, will not affect the estimation capability in TRAM. The total CPU time to integrate the state vector, the transition matrix and its inverse 1500 seconds was two minutes and 32 seconds. This is an acceptable execution time.

5.2 Verification of the Accuracy of the Numerical Partials

The transition matrix is initialized to the identity matrix every five seconds and then integrated for a five second span. To bound the errors caused by the use of the numerical partials, this was done twice, once using analytic partials and once using numerical partials, and the difference between the two transition matrices was computed. This difference matrix is then multiplied by a vector which has as its elements the largest error which can occur at any step in position and velocity. The product of the matrix times the vector gives the maximum error that can occur at each step caused by the use of numerical partials. If the resulting vector is multipled by the number of steps, a bound for the total error due to numerical partials is obtained.

In the error matrix which was obtained by subtracting the transition matrix computed using analytic partials from the transition matrix using numerical partials no term was larger than 10^{-14}. This meant that the terms which propagates position and velocity errors into position and the terms which propagate position and velocity errors into velocity were all less than 10^{-14}. Further, the maximum TRAM estimation error which can be made in position at any step is less than 100 feet and in velocity it is one foot per second. Since the data could be taken at one tenth second for 1500 seconds, their would be 15000 steps. These numbers provide the following bounds for the errors caused by the use of numerical partials.

$$\begin{pmatrix} \varepsilon_{1p} \\ \\ \varepsilon_{1v} \end{pmatrix} \leq 15000 \times \begin{pmatrix} 10^{-14} & 10^{-14} \\ \\ 10^{-14} & 10^{-14} \end{pmatrix} \times \begin{pmatrix} 10^{2} \\ \\ 1 \end{pmatrix}$$

$$\varepsilon_{1p} \leq 15.15 \times 10^{-9} \text{ feet}$$

$$\varepsilon_{1v} \leq 15.15 \times 10^{-9} \text{ feet/sec}$$

where ε_{1p} is the error in position caused by the use of numerical partials and ε_{1v} in the error in velocity caused by the use of numerical partials. Clearly, these errors are insignificant.

5.3 Verification of Accuracy of Spline Interpolation

To determine what the accuracy of the Spline interpolation was, the trajectory was integrated to 1500 seconds. At the beginning, at mid trajectory and at the end of the trajectory, Spline polynomials were fit over the five second interval. The state vector, the transition matrix and its inverse were approximated when analytic and also when numerical partials were used.

The error in position in the state vector was always less than 10^{-4} feet and the error in velocity was less than 10^{-5} feet per second. In the transition matrix, the term which propagates errors in position into position is less than 10^{-9}, and the term which propagates velocity error into position is less than 10^{-7}. The term which propagates position error into velocity was less than 10^{-11}, and the term which propagates velocity error into

velocity was 10^{-9}. Since position estimation errors are less than 100 feet per step, and velocity estimation errors are less than one foot per step and there are 15000 steps, the error terms are bounded by

$$\begin{pmatrix} \varepsilon_{2p} \\ \varepsilon_{2v} \end{pmatrix} \leq \begin{pmatrix} 10^{-9} & 10^{-7} \\ 10^{-11} & 10^{-9} \end{pmatrix} \times \begin{pmatrix} 10^{2} \\ 1 \end{pmatrix} \times 15000$$

where ε_{2p} and ε_{2v} are errors in position and velocity respectively which are caused because of Spline interpolation.

Therefore

$$\varepsilon_{2p} \leq 3 \times 10^{-3} \text{ feet}$$

$$\varepsilon_{2v} \leq 3 \times 10^{-5} \text{ feet/sec.}$$

5.4 Numerical Error Summary

For the state vector, the two sources of error, one from numerical integration and the other from Spline interpolation cause the following total error

$$\eta_p \leq 10^{-3} + 10^{-4} = .0011 \text{ feet}$$

$$\eta_v \leq 10^{-5} + 10^{-5} = 2 \times 10^{-5} \text{ feet/second}$$

where η_p and η_v are position and velocity errors because of numerical approximation. Both of these errors are insignificant.

For the transition matrix the following total error bounds are valid.

$$\varepsilon_{pt} \leq \varepsilon_{1p} + \varepsilon_{2p} = 2 \times 10^{-8} + 3 \times 10^{-3} < 3.1 \times 10^{-3}$$

$$\varepsilon_{vt} \leq \varepsilon_{1v} + \varepsilon_{2v} = 2 \times 10^{-8} + 3 \times 10^{-5} < 3.1 \times 10^{-5}$$

Both of these errors are insignificant. It may be concluded, therefore, that the errors due to the numerical approximations discussed here will in no way impair the estimation accuracy in TRAM and further, it may be concluded that these algorithms will greatly improve the speed and flexibility of the computation of the estimation.

VI DOCUMENTATION OF SUBROUTINES

6.1 GENERAL COMMENTS ON DOCUMENTATION

The documentation which follows is intended to be sufficient enough so that the capabilities of these subroutines can be understood, used and modified. The function that the subroutine performs is stated first. This is followed by a description of the input and output variables. The restrictions, if any, under which the subroutine must be used come next, and then a list of the subroutines this subroutine calls is given. The documentation also includes a listing of the subroutine and a flow chart.

The main subroutines are INTCON and INTERP, but these call others which are documented also.

6.2 INTCON SUBROUTINE DESCRIPTION

FUNCTION: Integrates the state vector, the transition matrix and its inverse over the interval of integration and then computes the coefficients for the interpolating Spline polynomials. A quintic Spline is determined for the state vector and a cubic Spline is determined for the transition matrix and its inverse.

INPUT:

/VAR/

X	Nine dimensional state vector containing position, velocity and acceleration. Only position and velocity are needed as input to INTCON.
T	Time at the beginning of the interval of integration.
TS	Start time of the trajectory
H	Step size of the integration
ID	Flag to indicate whether or not the inverse of the transition matrix should be integrated.

OUTPUT:

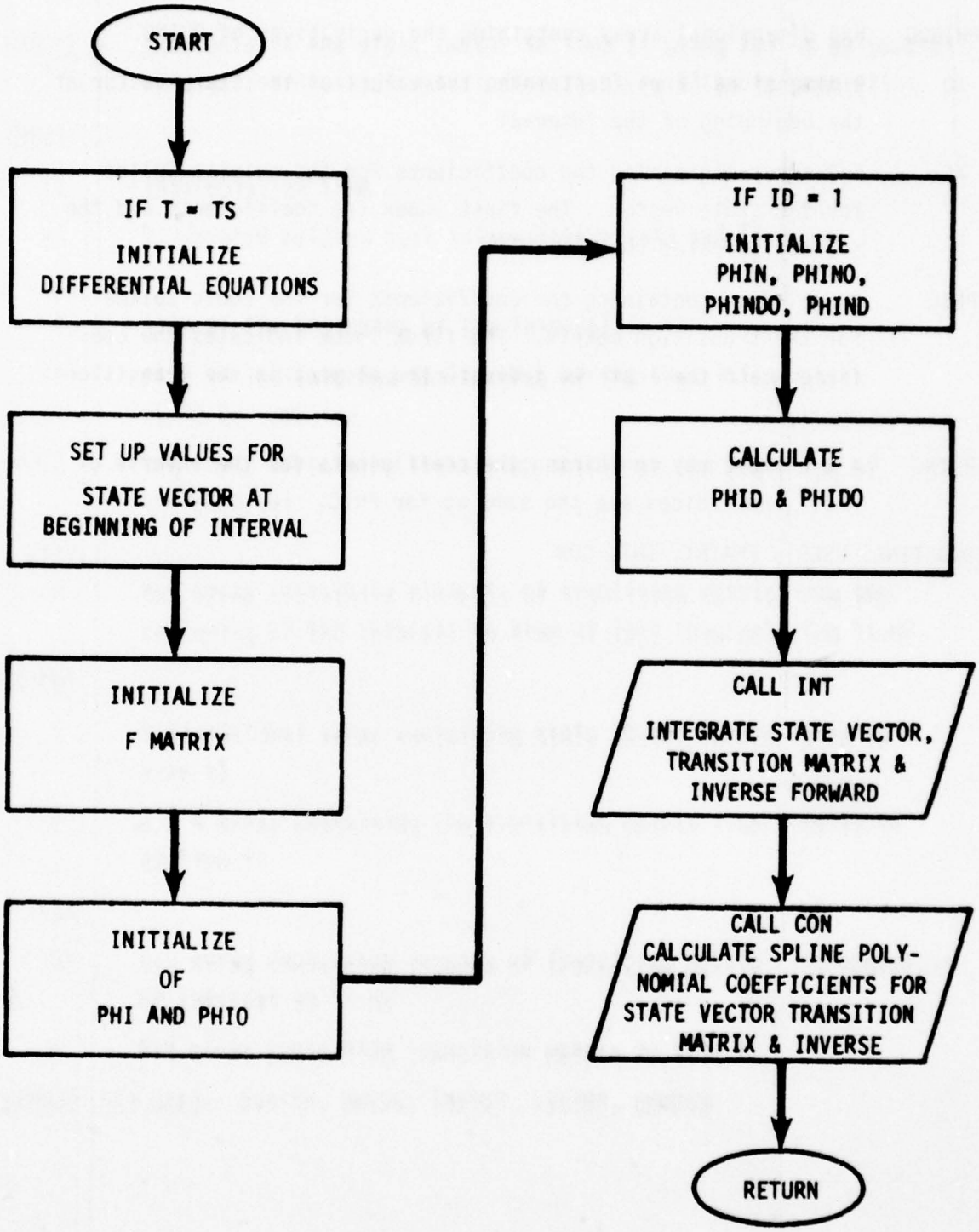
/VAR/

X	Nine dimentional state vector which contains position, velocity and acceleration integrated to the end of the interval
PHI	6x6 dimensional array containing elements of the transition matrix at the end of the interval of integration
PHID	6x6 dimensional array containing the derivatives of PHI
PHIN	6x6 dimensional array containing the elements of the inverse of PHI
PHIND	6x6 dimensional array containing the derivatives of PHIN
PHIO	6x6 dimensional array containing the elements of the transition matrix at the beginning of the interval of integration
PHIDO	6x6 dimensional array containing the derivatives of the elements of PHIO

PHINO	6x6 dimensional array containing the inverse of PHIO
PHINDO	6x6 dimensional array containing the derivatives of PHINO
XO	9 dimensional array containing the values of the state vector at the beginning of the interval
XC	6x3 array containing the coefficients for the quintic Spline for the state vector. The first index the coefficients and the second indicates the variable.
PHIC	4x6x6 array containing the coefficients for the cubic Spline for the transition matrix. The first index indicates the coefficient and the last two indicate the element in the transition matrix
PHINC	4 x 6 x 6 array containing the coefficients for the inverse of PHI. The indices are the same as for PHIC.

SUBROUTINES USED: FMATRX, INT, CON

INTCON FLOWCHART



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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE INTCON
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/VAR/X(9),PHI(6,6),PHID(6,6),PHIN(6,6),PHIND(6,6),
                      1 PHIO(6,6),PHIDO(6,6),PHINO(6,6),PHINDO(6,6),XO(9),XC(6,3),
                      2 PHIC(4,6,6),PHINC(4,6,6),TS,T,H,ID
   ISN 0005            COMMON/PHIM/F(6,6)
   ISN 0006            DIMENSION DX(6)
   ISN 0007            IF(T.NE.TS) GO TO 5
   ISN 0009            CALL DIFFUN(6,T,X,DX)
   ISN 0010            DO 3 I=1,3
   ISN 0011          3 X(I+6)=DX(I+3)
   ISN 0012          5 CONTINUE
   ISN 0013            DO 10 I=1,9
   ISN 0014         10 XO(I)=X(I)
   ISN 0015            CALL FMATRX(X)
   ISN 0016            DO 20 I=1,6
   ISN 0017            DO 20 J=1,6
   ISN 0018            PHI(I,J)=0D0
   ISN 0019            IF(I.EQ.J) PHI(I,J)=1D0
   ISN 0021            PHIO(I,J)=PHI(I,J)
   ISN 0022         20 CONTINUE
   ISN 0023            IF(ID.EQ.0) GO TO 19
   ISN 0025            DO 22 I=1,6
   ISN 0026            DO 22 J=1,6
   ISN 0027            PHIN(I,J)=PHI(I,J)
   ISN 0028            PHINO(I,J)=PHI(I,J)
   ISN 0029      22    CONTINUE
   ISN 0030            DO 18 J=1,6
   ISN 0031            DO 18 I=1,6
   ISN 0032            PHINDO(I,J)=0D0
   ISN 0033            DO 17 K=1,6
   ISN 0034         17 PHINDO(I,J)=-PHINO(I,K)*F(K,J)+PHINDO(I,J)
   ISN 0035         18 PHIND(I,J)=PHINDO(I,J)
   ISN 0036         19 CONTINUE
   ISN 0037            CALL DPHI(PHI,PHID)
   ISN 0038            DO 25 I=1,6
   ISN 0039            DO 25 J=1,6
   ISN 0040         25 PHIDO(I,J)=PHID(I,J)
   ISN 0041            CALL INT(X,DX,PHI,PHID,PHIN,PHIND,T,H,ID)
   ISN 0042            CALL CON
   ISN 0043            RETURN
   ISN 0044            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      43 ,PROGRAM SIZE =     1132

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          67K BYTES OF CORE NOT USED
```

6.3 INTERP SUBROUTINE DESCRIPTION

FUNCTION: Interpolates the state vector to time TI using Spline polynomials, obtains a transition matrix form TI-HS to TI by interpolation

INPUT:

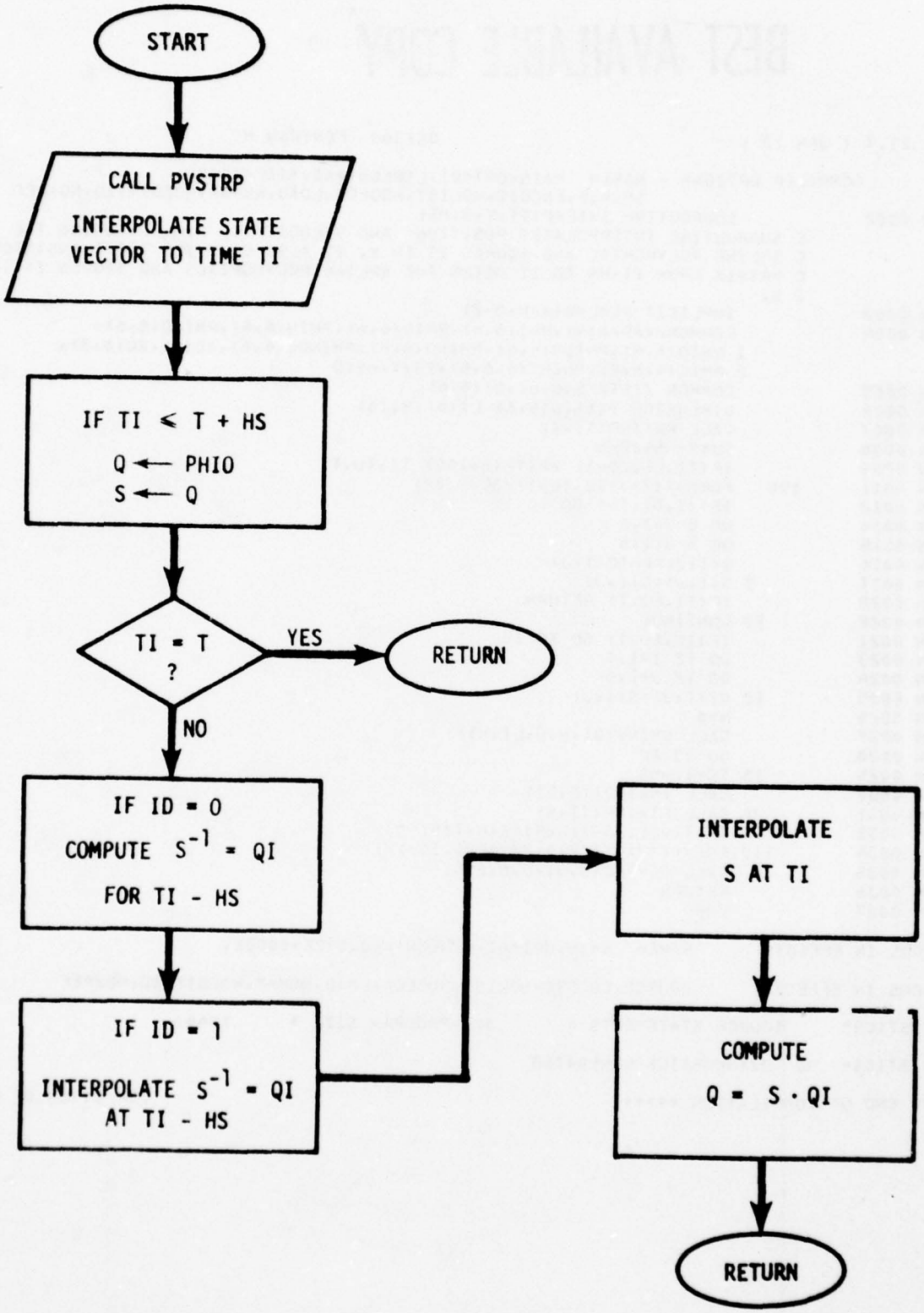
TI	Interpolation time
HS	Difference between last interpolation time and TI
/VAR/	
T	Time at the beginning of the interval
ID	Flag to indicate how the inverse of the transition matrix is to be computed
PHIO	6x6 array containing transition matrix at the beginning of the interval
/TST/	
S	6x6 array containing elements of transition matrix from the beginning of the interval to time of last interpolation TI-HS

OUTPUT:

Y	6 dimensional array containing state vector interpolated to time TI
Q	6 x 6 array containing the transition matrix from time TI-HS to time TI
/TST/	
QI	6x6 array containing inverse of transition matrix from beginning of interval to TI-HS
S	6x6 array containing transition matrix to time TI

SUBROUTINES USED: PVXTRP, DMINV, INXTRP, FIXTRP, DGMPRD

INTERP FLOWCHART



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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                              DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE INTERP(TI,Y,Q,HS)
                     C SUBROUTINE INTERPOLATES POSITION  AND VELOCITY TO TIME TI USING THE
                     C SPLINE POLYNOMIAL AND STORES IT IN Y. IT ALSO COMPUTES THE TRANSITION
                     C MATRIX FROM TI-HS TO TI USING THE SPLINE POLYNOMIALS AND STORES IT IN
                     C Q.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/VAR/X(9),PHI(6,6),PHID(6,6),PHIN(6,6),PHINO(6,6),
                      1 PHIO(6,6),PHIDO(6,6),PHINO(6,6),PHINDO(6,6),XO(9),XC(6,3),
                      2 PHIC(4,6,6),PHINC(4,6,6),TS,T,H,ID
   ISN 0005            COMMON /TST/ S(6,6),QI(6,6)
   ISN 0006            DIMENSION Y(6),Q(6,6),L1(6),M1(6)
   ISN 0007            CALL PVXTRP(TI,Y)
   ISN 0008            TU=T+HS+1D-6
   ISN 0009            IF(TI.LE.2D-1) WRITE(6,100) TI,TU,T
   ISN 0011     100    FORMAT(2X,'TI,TU,T',3E20.8/)
   ISN 0012            IF(TI.GT.TU) GO TO 10
   ISN 0014            DO 5 I=1,6
   ISN 0015            DO 5 J=1,6
   ISN 0016            Q(I,J)=PHIO(I,J)
   ISN 0017          5 S(I,J)=Q(I,J)
   ISN 0018            IF(TI.EQ.T) RETURN
   ISN 0020         10 CONTINUE
   ISN 0021            IF(ID.EQ.1) GO TO 15
   ISN 0023            DO 12 I=1,6
   ISN 0024            DO 12 J=1,6
   ISN 0025         12 QI(I,J)=S(I,J)
   ISN 0026            N=6
   ISN 0027            CALL DMINV(QI,N,D,L1,M1)
   ISN 0028            GO TO 20
   ISN 0029         15 TE=TI-HS
   ISN 0030            CALL INXTRP(TE,QI)
   ISN 0031         20 CALL FIXTRP(TI,S)
   ISN 0032            IF(TI.LE.2D-1) WRITE(6,110) S
   ISN 0034        110 FORMAT(2X,'S'/,6(2X,6E20.10/)/)
   ISN 0035            CALL DGMPRD(S,QI,Q,6,6,6)
   ISN 0036            RETURN
   ISN 0037            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      36 ,PROGRAM SIZE =     1046

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  75K BYTES OF CORE NOT USED
```

6.4 INT SUBROUTINE DESCRIPTION

FUNCTION: Integrates state vector, transition matrix and its inverse from T to T+H

INPUT:

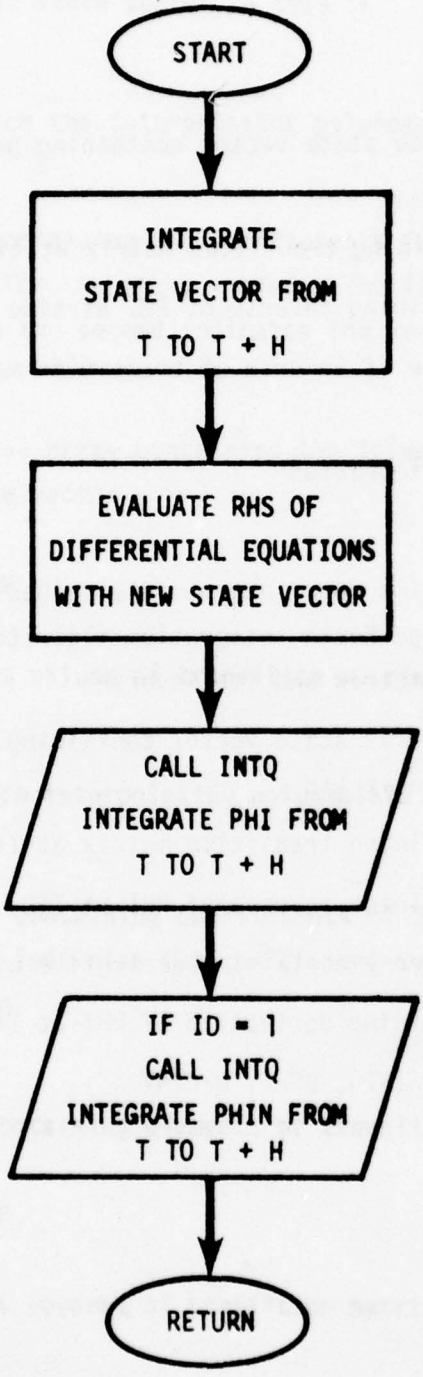
X	9 dimension array state vector containing position, velocity and acceleration
PHI	6x6 array containing transition matrix at time T
PHIN	6x6 array containing inverse of PHI at time T
ID	Flag to indicate if inverse of transition matrix should be integrated
H	Step size of integration

OUTPUT:

PS	6x4 array contains four intermediate values of the state vector used in the Runge-Kutta integration algorithm for integrating the transition matrix and its inverse
X	9 dimensional array state vector containing position, velocity and acceleration at T+H
PHI	6x6 array containing transition matrix at T+H
PHIN	6x6 array containing inverse of PHI at T+H
DX	6 dimensional array containing derivative of X at T+H
PHID	6x6 array containing derivative of PHI at T+H

SUBROUTINES USED: DIFFUN, INTQ, DPHI, DPHINV

INT FLOWCHART



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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE INT(X,DX,PHI,PHID,PHIN,PHIND,T,H,ID)
                     C SUBROUTINE INT GIVES THE VALUES OF POSITION AND VELOCITY. THE
                     C TRANSITION MATRIX AND ITS DERIVATIVE AND, AS AN OPTION, THE INVERSE
                     C OF THE TRANSITION AND ITS DERIVATIVE AT TIME T+H. ID IS A FLAG WHICH
                     C INDICATES WHETHER THE INVERSE OF THE TRANSITION MATRIX AND ITS
                     C DERIVATE IS TO BE COMPUTED. THE POSITION AND VELOCITY ARE STORED IN X.
                     C THE TRANSITION MATRIX AND ITS INVERSE ARE STORED IN PHI AND PHID.
                     C THE INVERSE OF TRANSITION MATRIX AND ITS DERIVATIVE ARE STORED IN
                     C PHIN AND PHIND.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            DIMENSION X(1),DX(1),PHI(6,6),PHID(6,6),PHIN(6,6),PHIND(6,6)
                      &PV(6),PS(6,4),Y(6),Q(6),DY(6),
                      & C(3),D(4)
   ISN 0005            EXTERNAL DPHI,DPHINV
   ISN 0006            DATA C/2*0.500,1D0 /,D/1D0,2*2D0,1D0/
   ISN 0007            DO 5 I=1,6
   ISN 0008            PV(I)=X(I)
   ISN 0009          5 PS(I,1)=X(I)
   ISN 0010            N=6
   ISN 0011            DO 30 I=1,4
   ISN 0012            IF(I.EQ.1) GO TO 15
   ISN 0014            K=I-1
   ISN 0015            DO 10 J=1,6
   ISN 0016            PV(J)=PS(J,1)+C(K)*Q(J)
   ISN 0017         10 PS(J,I)=PV(J)
   ISN 0018         15 CONTINUE
   ISN 0019            CALL DIFFUN(N,T,PV,DY)
   ISN 0020            DO 20 J=1,6
   ISN 0021         20 Q(J)=H*DY(J)
   ISN 0022            DO 25 J=1,6
   ISN 0023         25 X(J)=X(J)+Q(J)*D(I)/6D0
   ISN 0024         30 CONTINUE
   ISN 0025            CALL DIFFUN(N,T,X,DX)
   ISN 0026            DO 31 I=1,3
   ISN 0027         31 X(I+6)=DX(I+3)
   ISN 0028            CALL INTQ(PS,PHI,PHID,T,H,X,DPHI)
   ISN 0029            IF(ID.EQ.1) CALL INTQ(PS,PHIN,PHIND,T,H,X,DPHINV)
   ISN 0031            RETURN
   ISN 0032            END

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       31 ,PROGRAM SIZE =      1464

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  75K BYTES OF CORE NOT USED
```

6.5 PVXTRP SUBROUTINE DESCRIPTION

FUNCTION: Interpolates state vector to time TE

INPUT:

TE — Time at which the interpolating polynomial is to be evaluated.

/VAR/

XC — 6x3 array containing the coefficients for the Spline interpolating polynomial for the state vector. The first indicates the coefficient and the second indicates the variable being interpolated.

OUTPUT:

Y — 6 dimentional array containing the interpolated values of position of the state vector

ENTRY FIXTRP

FUNCTION: Interpolates values of transition matrix to time TE

INPUT:

TE — Time at which interpolating polynomials are to be evaluated

/VAR/

PHIC — 4x6x6 array containing coefficients of Spline polynomials. The first index indicates the coefficient and the last two indicate the variable

OUTPUT:

Q — 6x6 array containing elements of transition matrix at time TE

ENTRY INXTRP

FUNCTION: Interpolates inverse of transition matrix to time TE

INPUT:

TE — Time at which the interpolating polynomial is to evaluated

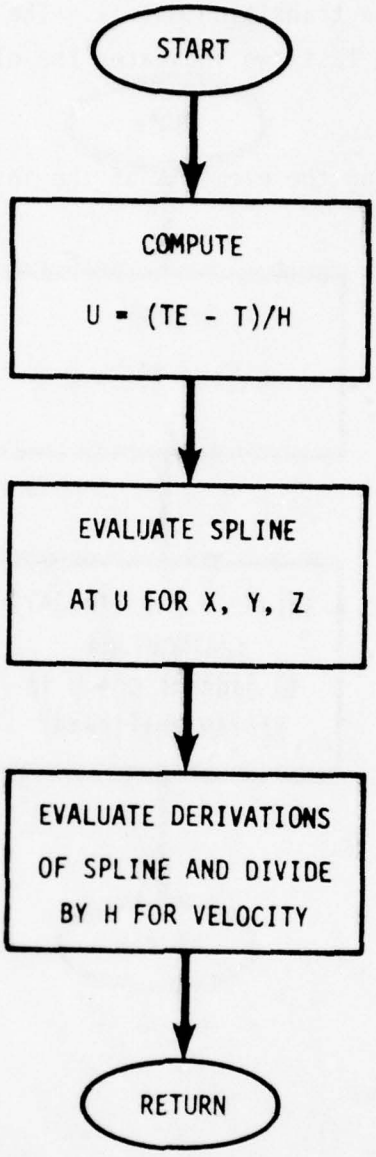
/VAR/

PHINC 4x6x6 array containing coefficients of the Spline polynomials for the inverse of the transition matrix. The first index indicates the array and the last two indicates the element.

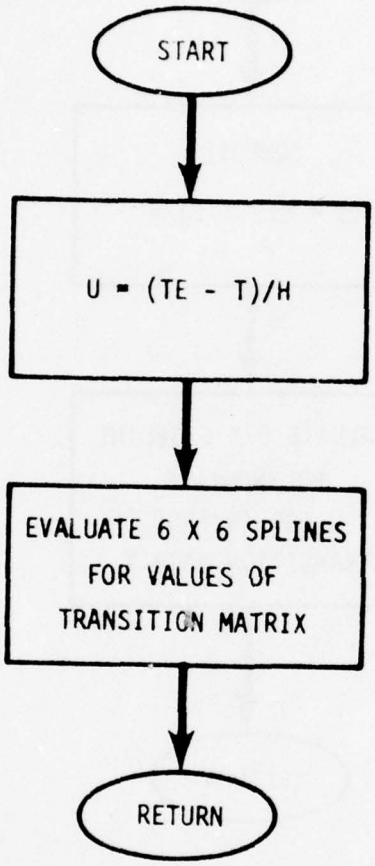
OUTPUT:

S 6x6 array containing the elements of the inverse of the transition matrix for time TE

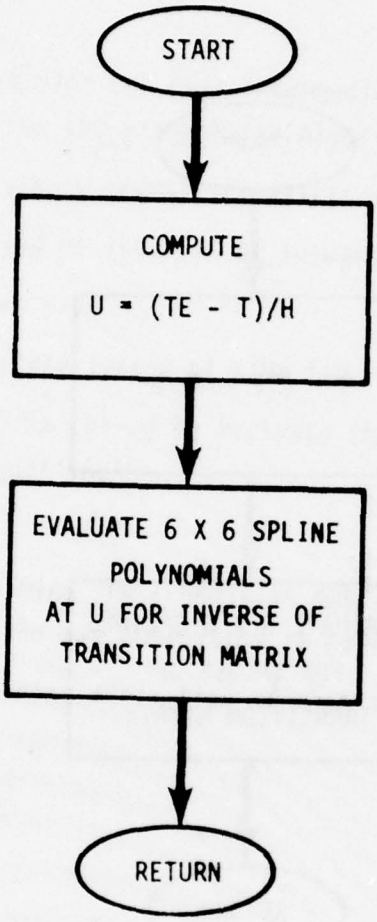
PVXTRP FLOWCHART



ENTRY FIXTRP



ENTRY INXTRP



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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                              DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE PVXTRP(TE,Y)
                     C SUBROUTINE PVXTRP(TE,Y) EXTRAPOLATED POSITION AND VELOCITY TO TIME TE
                     C USING A QUINTIC SPLINE FOR POSITION AND ITS DERIVATIVE FOR VELOCITY.
                     C THE COEFFICIENTS FOR THE QUINTIC SPLINE ARE COMPUTED IN SUBROUTINE
                     C INTCON. INTERPOLATED POSITION AND VELOCITY ARE STORED IN Y.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/VAR/X(9),PHI(6,6),PHID(6,6),PHIN(6,6),PHIND(6,6),
                      1 PHIO(6,6),PHIDO(6,6),PHINO(6,6),PHINDO(6,6),XO(9),XC(6,3),
                      2 PHIC(4,6,6),PHINC(4,6,6),TS,T,H,ID
   ISN 0005            DIMENSION Q(6,6)
   ISN 0006            DIMENSION S(6,6)
   ISN 0007            DIMENSION Y(6)
   ISN 0008            U=(TE-T)/H
   ISN 0009            DO 10 I=1,3
   ISN 0010            J=I+3
   ISN 0011            Y(I)= U*U*U*U*U*XC(1,I)+U*U*U*U*XC(2,I)+U*U*U*XC(3,I)+U*U*XC(4,I)
                      1 +U*XC(5,I)+XC(6,I)
   ISN 0012            Y(J)=5D0*U*U*U*U*XC(1,I)+4D0*U*U*U*XC(2,I)+3D0*U*U*XC(3,I)
                      1 +2D0*U*XC(4,I)+XC(5,I)
   ISN 0013            Y(J)=Y(J)/H
   ISN 0014         10 CONTINUE
   ISN 0015            RETURN
   ISN 0016            ENTRY FIXTRP(TE,Q)
                     C SUBROUTINE FIXTRP(TE,Q) EXTRAPOLATES PHI TO TIME TE USING A CUBIC
                     C SPLINE. THE COEFFICIENTS FOR THE CUBIC SPLINE ARE COMPUTED IN
                     C SUBROUTINE INTCON. INTERPOLATED VALUES OF PHI ARE STORED IN Q.
   ISN 0017            U=(TE-T)/H
   ISN 0018            DO 15 I=1,6
   ISN 0019            DO 15 J=1,6
   ISN 0020         15 Q(I,J)=U*U*U*PHIC(1,I,J)+U*U*PHIC(2,I,J)+U*PHIC(3,I,J)+PHIC(4,I,J)
   ISN 0021            RETURN
   ISN 0022            ENTRY INXTRP(TE,S)
                     C SUBROUTINE INXTRP(TE,S) EXTRAPOLATES THE INVERSE OF PHI TO TIME TE
                     C USING A CUBIC SPLINE. THE COEFFICIENTS FOR THE CUBIC SPLINE ARE
                     C COMPUTED IN SUBROUTINE INTCON. INTERPOLATED VALUES OF PHI INVERSE ARE
                     C STORED IN S.
   ISN 0023            U=(TE-T)/H
   ISN 0024            DO 20 I=1,6
   ISN 0025            DO 20 J=1,6
   ISN 0026         20 S(I,J)=U*U*U*PHINC(1,I,J)+U*U*PHINC(2,I,J)+U*PHINC(3,I,J)+
                      2 PHINC(4,I,J)
   ISN 0027            RETURN
   ISN 0028            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       27 ,PROGRAM SIZE =     1354

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  71K BYTES OF CORE NOT USED
```

6.6 INTQ SUBROUTINE DESCRIPTION

FUNCTION: Integrates both the transition matrix and its inverse from T to T+H. [The subroutine for evaluating the differential equations is passed in via the calling list.]

INPUT:

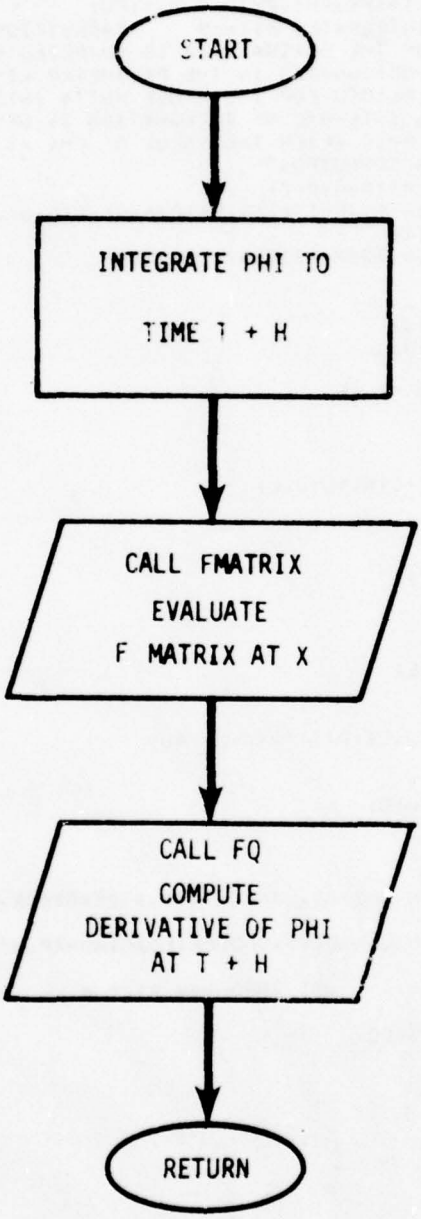
PS	6x4 array containing the four intermediate values of the state vector needed for the Runge-Kutta algorithm
PHI	6x6 array containing transition matrix or inverse at time T
T	Time at beginning of interval of integration
H	Integration step size
X	9 dimensional state vector at time T+H
FQ	Procedure which is called to evaluate the right hand side of the differential equations

OUTPUT:

PHI	6x6 array containing the transition matrix of its inverse, dependent upon the way subroutine is called, at T+H
PHID	6x6 array containing derivative of what is in PHI at time T+H

SUBROUTINES USED: FMATRX, FQ

INTQ FLOWCHART



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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE INTQ(PS,PHI,PHID,T,H,X,FQ)
                     C SUBROUTINE INTQ INTEGRATES EITHER     TRANSITION MATRIX FROM T TO T+H.
                     C THE COMPUTATION OF THE DERIVATIVES IS SUPPLIED BY AN EXTERNAL
                     C PROCEDURE AND IS DESIGNATED IN THE PARAMETER LIST AS FQ. PS ARE THE
                     C FOUR VALUES OF X NEEDED FOR THE RUNGE KUTTA INTEGRATION ALGORITHM. THE
                     C VALUE OF X AT THE INTERVAL OF INTEGRATION IS DESIGNATED AS X. IT IS
                     C USED TO EVALUATE PHID AFTER THE VALUE OF PHI AT THE END OF THE
                     C INTERVAL HAS BEEN COMPUTED.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            DIMENSION PS(6,4),PHI(6,6),PHID(6,6),S(6,6),Q(6,6),U(6,6),Y(6),
                      & C(3),D(4),X(1)
   ISN 0005            DATA C/2*0.5D0,1D0/,D/1D0,2*2D0,1D0/
   ISN 0006            DO 5 I=1,6
   ISN 0007            DO 5 J=1,6
   ISN 0008            U(I,J)=PHI(I,J)
   ISN 0009          5 S(I,J)=PHI(I,J)
   ISN 0010            DO 30 I=1,4
   ISN 0011            IF(I.EQ.1) GO TO 15
   ISN 0013            K=I-1
   ISN 0014            DO 10 J=1,6
   ISN 0015            DO 10 L=1,6
   ISN 0016         10 U(J,L)=S(J,L)+C(K)*Q(J,L)
   ISN 0017         15 CONTINUE
   ISN 0018            DO 16 J=1,6
   ISN 0019         16 Y(J)=PS(J,I)
   ISN 0020            CALL FMATRX(Y)
   ISN 0021            CALL FQ(U,Q)
   ISN 0022            DO 17 J=1,6
   ISN 0023            DO 17 L=1,6
   ISN 0024         17 Q(J,L)=H*Q(J,L)
   ISN 0025            DO 25 J=1,6
   ISN 0026            DO 25 L=1,6
   ISN 0027         25 PHI(J,L)=PHI(J,L)+D(I)*Q(J,L)/6D0
   ISN 0028         30 CONTINUE
   ISN 0029            CALL FMATRX(X)
   ISN 0030            CALL FQ(PHI,PHID)
   ISN 0031            RETURN
   ISN 0032            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       31 ,PROGRAM SIZE =     2008

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          71K BYTES OF CORE NOT USED
```

6.7 DPHINV SUBROUTINE DESCRIPTION

FUNCTION: Computes derivative of the inverse of the transition matrix

INPUT:

Q 6x6 array containing inverse of transition matrix

/FHIM/

F 6x6 array containing coefficients of differential equations for inverse of transition matrix

OUTPUT:

DQ 6x6 array containing derivatives of the inverse of the transition matrix

ENTRY DPHI

FUNCTION: Computes derivatives of transition matrix

INPUT:

Q 6x6 array containing transition matrix

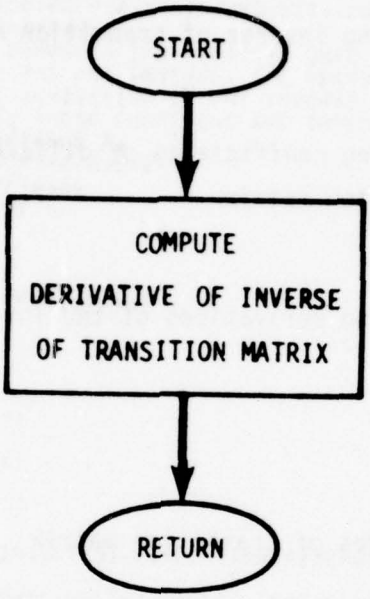
/PHIM/

F 6x6 array containing coefficients of differential equations for transition matrix

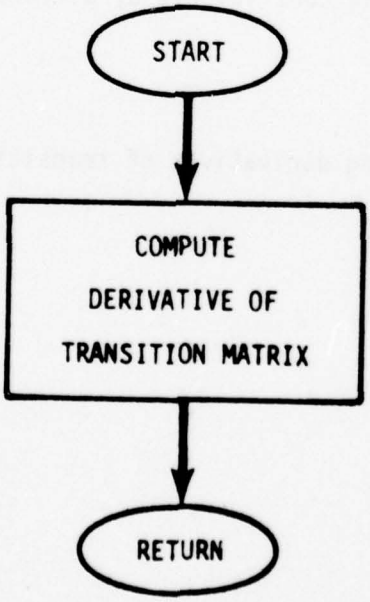
OUTPUT:

DQ 6x6 array containing derivatives of transition matrix

DPHINV FLOWCHART



ENTRY DPHI



```
LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE DPHINV(Q,DQ)
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/PHIM/F(6,6)
   ISN 0005            DIMENSION Q(6,6),DQ(6,6)
   ISN 0006            DO 10 I=1,6
   ISN 0007            DO 10 J=1,6
   ISN 0008            DQ(I,J)=0D0
   ISN 0009            DO 10 K=1,6
   ISN 0010         10 DQ(I,J)=DQ(I,J)-Q(I,K)*F(K,J)
   ISN 0011            RETURN
   ISN 0012            ENTRY DPHI(Q,DQ)
   ISN 0013            DO 20 I=1,6
   ISN 0014            DO 20 J=1,6
   ISN 0015            DQ(I,J)=0D0
   ISN 0016            DO 20 K=1,6
   ISN 0017         20 DQ(I,J)=DQ(I,J)+F(I,K)*Q(K,J)
   ISN 0018            RETURN
   ISN 0019            END

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       18 ,PROGRAM SIZE =       800

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  79K BYTES OF CORE NOT USED
```

6.8 FMATRX SUBROUTINE DESCRIPTION

FUNCTION: Computes matrix which relates transition matrix and its inverse to their derivatives

INPUT:

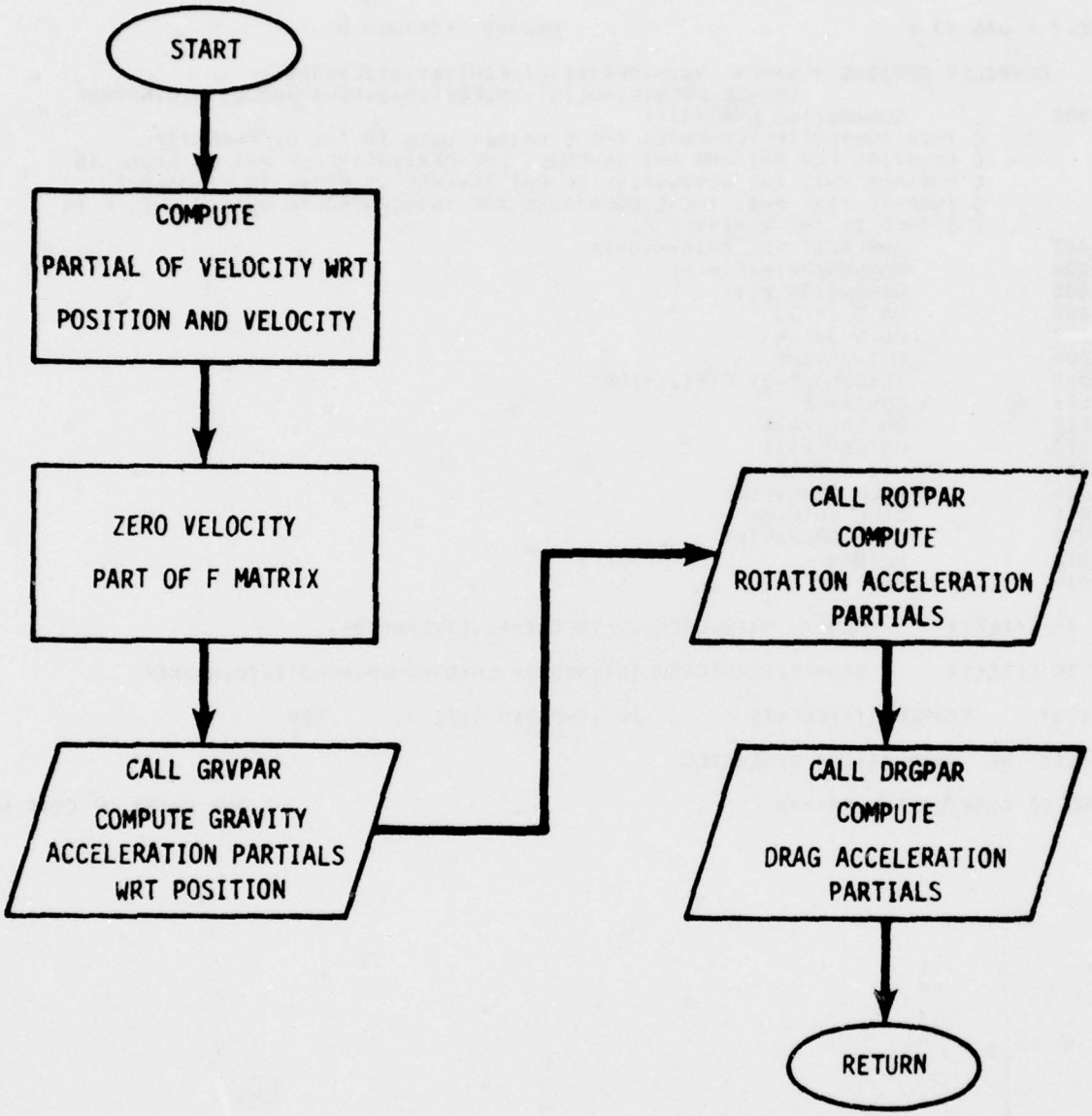
X 9 dimensional array containing position, velocity and acceleration

OUTPUT:

F 6x6 dimensional array containing matrix which relates transition matrix and its inverse to their derivatives,

SUBROUTINES USED: GRVPAR, ROTPAR, DRGPAR

FMATRX FLOWCHART



```
LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE FMATRX(X)
                     C THIS SUBROUTINE COMPUTES THE F MATRIX USED IN THE DIFFERENTIAL
                     C EQUATION FOR PHI AND PHI INVERSE. THE DERIVATIVE OF PHI IS EQUAL TO
                     C F TIMES PHI. THE DERIVATIVE OF PHI INVERSE IS EQUAL TO MINUS PHI
                     C INVERSE TIME PHI. THESE EQUATIONS ARE INTEGRATED TO OBTAIN PHI. F IS
                     C STORED IN THE VARIABLE F.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/PHIM/F(6,6)
   ISN 0005            DIMENSION X(9)
   ISN 0006            DO 5 I=1,3
   ISN 0007            DO 5 J=1,6
   ISN 0008            F(I,J)=0D0
   ISN 0009            IF(J.EQ.I+3) F(I,J)=1D0
   ISN 0011          5 CONTINUE
   ISN 0012            DO 10 I=4,6
   ISN 0013            DO 10 J=1,6
   ISN 0014         10 F(I,J)=0D0
   ISN 0015            CALL GRVPAR(X)
   ISN 0016            CALL ROTPAR
   ISN 0017            CALL DRGPAR(X)
   ISN 0018            RETURN
   ISN 0019            END

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       18 ,PROGRAM SIZE =      508

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          79K BYTES OF CORE NOT USED
```

6.9 DRGPAR SUBROUTINE DESCRIPTION

In this application this subroutine is a dummy subroutine. Drag is not modeled in either the powered flight or free fall segments of the trajectory. However, when the reentry portion of the trajectory is considered, these partials will have to be computed.

```
LEVEL 21.7 ( JAN 73 )                    05/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE DRGPAR(X)
                     C SUBROUTINE DRGPAR COMPUTES THE PARTIALS OF ACCELERATION DUE TO DRAG
                     C WITH RESPECT TO POSITION AND VELOCITY. THESE ARE ADDED INTO THE MATRIX
                     C F.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/PHIM/F(6,6)
   ISN 0005            DIMENSION X(9)
   ISN 0006            S=100
   ISN 0007            RETURN
   ISN 0008            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =        7 ,PROGRAM SIZE =      238

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          79K BYTES OF CORE NOT USED
```

6.10 ROTPAR SUBROUTINE DESCRIPTION

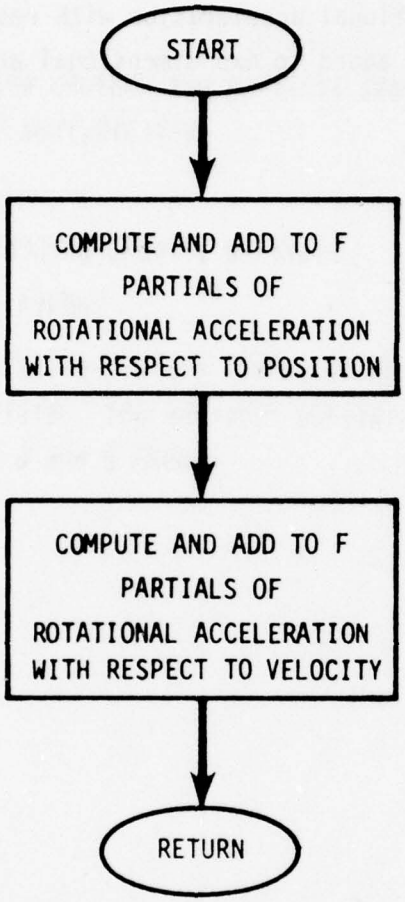
FUNCTION: Computes partials of acceleration due to the rotation of the coordinate system with respect to position and velocity

OUTPUT:

F Partials of rotational acceleration with respect to position and velocity are added to 6x6 dimensional array containing F

SUBROUTINES USED: None

ROTPAR FLOWCHART



```
LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                              DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE POTPAR
                     C SUBROUTINE POTPAR COMPUTES THE PARTIALS OF  ACCELERATION DUE TO THE
                     C ROTATION OF THE EARTH WITH RESPECT TO POSITION AND VELOCITY. THESE ARE
                     C ADDED INTO THE F MATRIX.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/PHIM/F(6,6)
   ISN 0005            DATA W/0.7291151D-4/
   ISN 0006            W2=W*W
   ISN 0007            F(4,1)=F(4,1)+W2
   ISN 0008            F(5,2)=F(5,2)+W2
   ISN 0009            F(4,5)=F(4,5)+W*2D0
   ISN 0010            F(5,4)=F(5,4)-W*2D0
   ISN 0011            RETURN
   ISN 0012            END

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       11 ,PROGRAM SIZE =       290

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                  79K BYTES OF CORE NOT USED
```

6.11 GRVPAR SUBROUTINE DESCRIPTION

FUNCTION: Computes partials of gravity acceleration with respect to position

INPUT:

X — 9 dimensional state vector. Only position terms are used

/TIG/

IN — Flag to indicate whether the partials are to be computed numerically or analytically

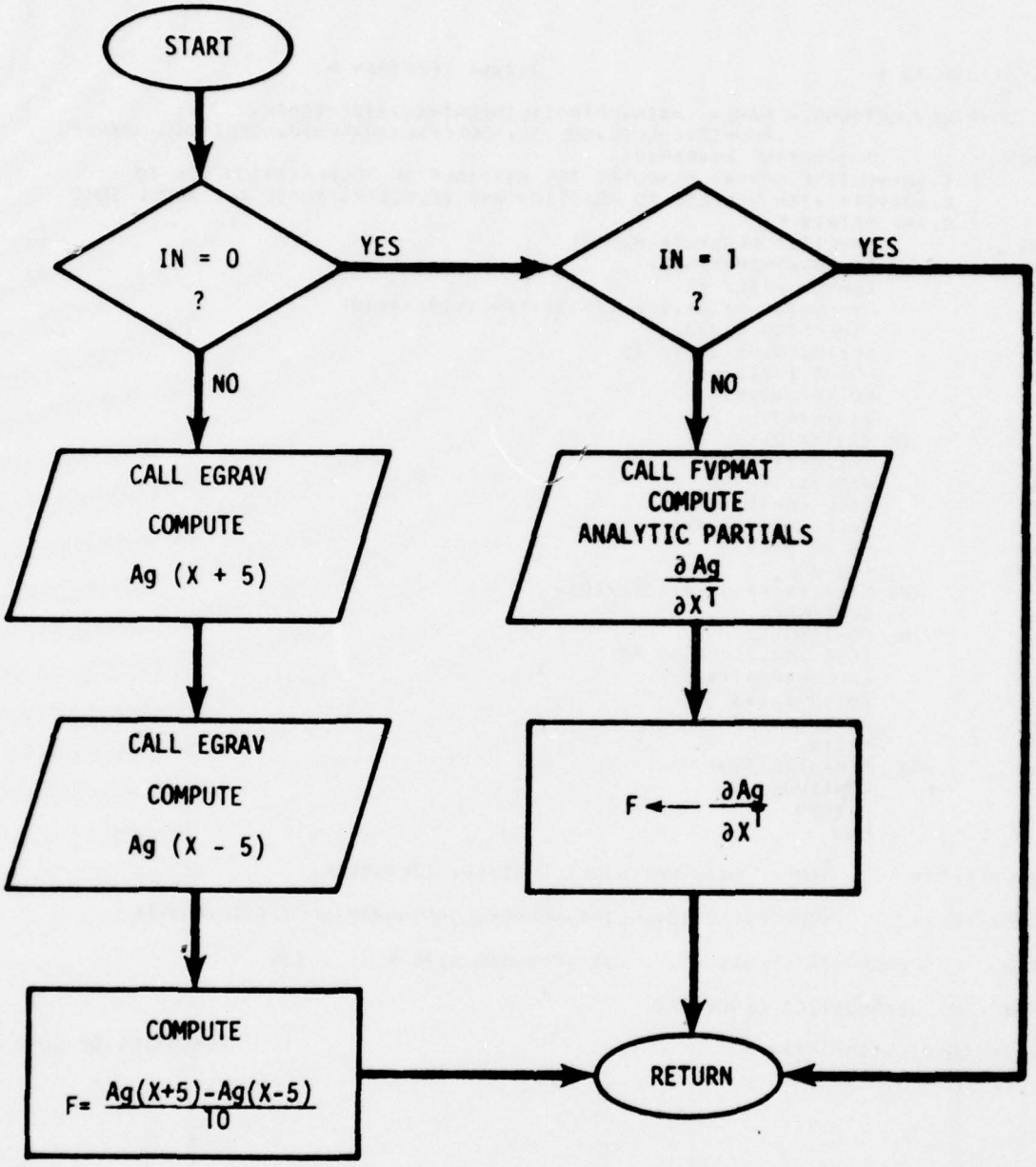
OUTPUT:

F — 6x6 array containing gravity partials

SUBROUTINES USED: EGRAV, FVPMAT

COMMENTS: GRVPAR must be called before ROTPAR or DRGPAR. IN = 0 gives numerical partials. The analytic partials and the gravity is based upon the J and D model.

GRVPAR FLOWCHART



```
LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE GRVPAR(X)
                     C SUBROUTINE GRVPAR COMPUTES THE PARTIALS OF ACCELERATION DUE TO
                     C GRAVITY WITH RESPECT TO POSITION AND VELOCITY. THESE ARE ADDED INTO
                     C THE MATRIX F.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON/PHIM/F(6,6)
   ISN 0005            COMMON /TIC/ IN
   ISN 0006            DIMENSION AX(3),Y(6),AY(3),X(9),V(9),AV(3)
   ISN 0007            DIMENSION G(3,3)
   ISN 0008            IF(IN.EQ.0) GO TO 70
   ISN 0010            DO 30 I=1,3
   ISN 0011            DO 10 J=1,6
   ISN 0012            V(J)=X(J)
   ISN 0013         10 Y(J)=X(J)
   ISN 0014            Y(I)=Y(I)+.500
   ISN 0015            V(I)=V(I)-.500
   ISN 0016            CALL EGRAV(Y,AY)
   ISN 0017            CALL EGRAV(V,AV)
   ISN 0018            DO 20 J=1,3
   ISN 0019            K=J+3
   ISN 0020         20 F(K,I)=(AY(J)-AV(J))/1D1
   ISN 0021         30 CONTINUE
   ISN 0022         70 CONTINUE
   ISN 0023            IF(IN.EQ.1) GO TO 60
   ISN 0025            CALL FVPMAT(X,G)
   ISN 0026            DO 50 I=1,3
   ISN 0027            DO 50 J=1,3
   ISN 0028            K=I+3
   ISN 0029         50 F(K,J)=G(I,J)
   ISN 0030         60 CONTINUE
   ISN 0031            RETURN
   ISN 0032            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       31 ,PROGRAM SIZE =      934

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          75K BYTES OF CORE NOT USED
```

6.12 EGRAV SUBROUTINE DESCRIPTION

FUNCTION: Computes gravity acceleration using J and D model of gravity

INPUT:

Y 9 dimensional array containing state vector. Only first 3 elements are used

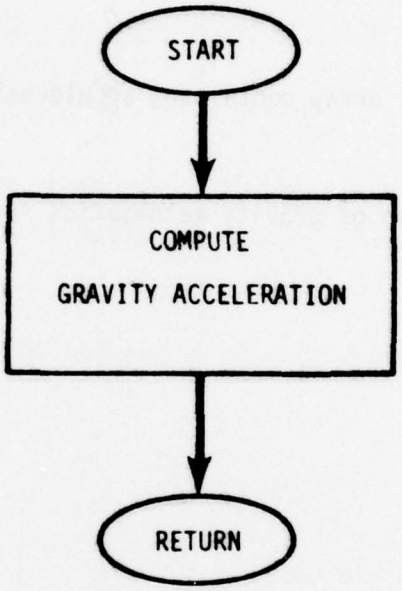
OUTPUT:

GY 3 dimensional array containing acceleration for three variables

SUBROUTINES USED: None

COMMENTS: For descripton of gravity mathematics see [4].

EGRAV FLOWCHART



```
LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE EGRAV(Y,GY)
   ISN 0003            IMPLICIT REAL*8 (A-H,O-Z)
   ISN 0004            DIMENSION Y(1),GY(1)
   ISN 0005            DATA GM,      COND,CONA,CJCAS/1.40764685 D16,
                      *     0.10279500 D-4, 20925672.6 D0, 0.711155219 D12/
                      C*************************************************************
                      C COMPUTE THE ACCELERATION IN EFG DUE TO GRAVITY
                      C* THIS SUBROUTINE IS TAKEN FROM A TRW REPORT ON IDT SIMULATION  *
                      C*************************************************************
   ISN 0006            A = CONA
   ISN 0007            D = COND
   ISN 0008            R2 = 0.
   ISN 0009            DO 10 I=1,3
   ISN 0010         10 R2 = R2 + Y(I)*Y(I)
   ISN 0011            R4 = R2*R2
   ISN 0012            Z2R2 = Y(3)*Y(3)/R2
   ISN 0013            A2 = A*A
   ISN 0014            DA4 = D*A2*A2
   ISN 0015            H = (GM/R2)*(1.D0 + (CJCAS/R2)*(1.D0 - 5.D0*Z2R2) + (DA4/R4)*
                      *     (0.42857143D0 - Z2R2*(6.D0 - 9.D0*Z2R2)))
   ISN 0016            A2 = DSQRT(R2)
   ISN 0017            GY(3) = -(Y(3)/A2)*(H + (GM/R4)*(2.D0*CJCAS + (DA4/R2)*
                      *          (1.71428571D0 - 4.D0*Z2R2)))
   ISN 0018            GY(1) = -(H*Y(1)/A2)
   ISN 0019            GY(2) = -(H*Y(2)/A2)
   ISN 0020            RETURN
   ISN 0021            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =       20 ,PROGRAM SIZE =      814

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          79K BYTES OF CORE NOT USED
```

6.13 DIFFUN SUBROUTINE DESCRIPTION

FUNCTION: Computes acceleration due to gravity and acceleration sensed because of rotating coordinate. Computes velocity of body in motion

INPUT:

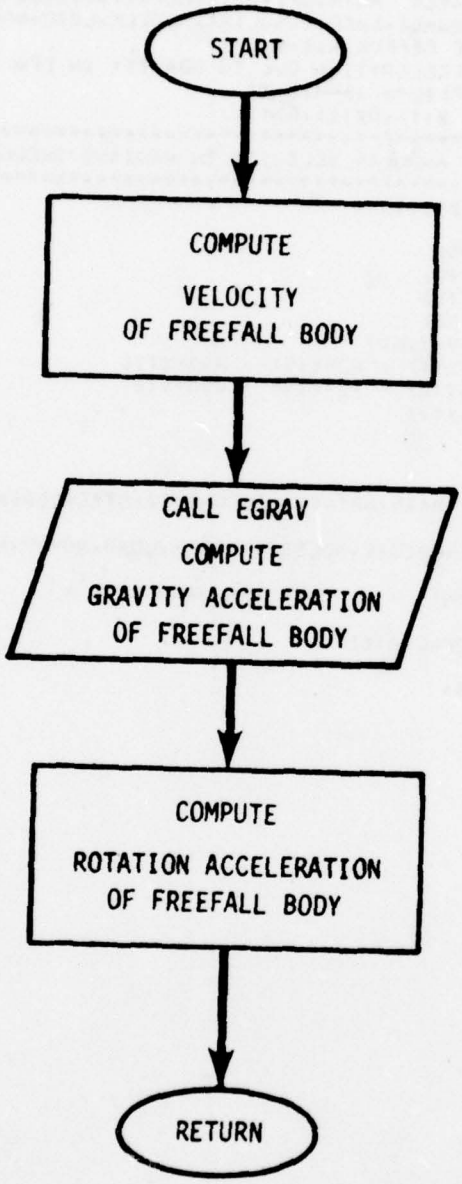
N	Number of differential equations (never used because it is always 6)
T	Time (not used)
Y	9 dimention array containing position, velocity and acceleration. Only position and velocity are used

OUTPUT:

DY	6 dimensional array containing gravity and rotation accelerations and velocities.

SUBROUTINES USED: EGRAV

DIFFUN FLOWCHART



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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                              DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE DIFFUN(N,T,Y,DY)
                     C COMPUTE THE ACCELERATION DUE TO GRAVITY IN EFG COORDINATES
   ISN 0003            IMPLICIT REAL*8 (A-H,O-Z)
   ISN 0004            DIMENSION Y(1),DY(1),GY(3)
                     C*****************************************************************
                     C* W IS EARTH'S ANGULAR VELOCITY IN RADIANS/SECOND                *
                     C*****************************************************************
   ISN 0005            W = 0.72921151D-4
   ISN 0006            WSQ = W*W
   ISN 0007            W2 = 2.D0*W
   ISN 0008         20 DY(1) = Y(4)
   ISN 0009            DY(2) = Y(5)
   ISN 0010            DY(3) = Y(6)
   ISN 0011            CALL EGRAV(Y,GY)
   ISN 0012            DY(4) = GY(1) + W2*Y(5) + WSQ*Y(1)
   ISN 0013            DY(5) = GY(2) - W2*Y(4) + WSQ*Y(2)
   ISN 0014            DY(6) = GY(3)
   ISN 0015            RETURN
   ISN 0016            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*      SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      15 ,PROGRAM SIZE =      496

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                          79K BYTES OF CORE NOT USED
```

6.14 FVPMAT SUBROUTINE DESCRIPTION

FUNCTION: Computes partials of gravity acceleration with respect to position

INPUT:

X — 9 dimensional array containing position, velocity and acceleration. Only position is used

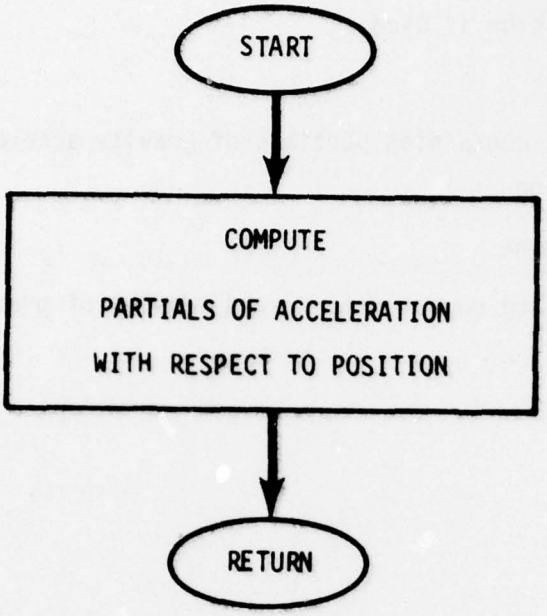
OUTPUT:

F — 3x3 array containing partials of gravity acceleration with respect to position

SUBROUTINES USED: None

COMMENTS: Partials are computed for J and D model of gravity

FVPMAT FLOWCHART



```
LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

         COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                            SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
  ISN 0002            SUBROUTINE PVPMAT(X,F)
                     C*********************************************************************
                     C*
                     C*    COMPUTE THE PARTIALS OF ACCELERATION WITH RESPECT TO POSITION
                     C*
                     C*    P1 = P( R, (1./R**2), (1./R**4) ) / P( X, Y, Z)
                     C*
                     C*    P2 = P( Z**2/R**2 ) / P( X, Y, Z)
                     C*
                     C*    P3 = P( X/R, Y/R, Z/R) / P( X, Y, Z)
                     C*
                     C*    P4 = P( H ) / P( X, Y, Z)
                     C*
                     C*********************************************************************
  ISN 0003            IMPLICIT REAL*8 (A-H,O-Z)
  ISN 0004            REAL*8 GM,JA2
  ISN 0005            DIMENSION X(1),P1(3,3),P2(3),P3(3,3),P4(3),F(3,3)
  ISN 0006            DATA COND/ 0.10279500 D-4 /
  ISN 0007            DATA CJCAS/ 0.711155219 D12 /
  ISN 0008            DATA W/ 0.72911510-4 /
  ISN 0009            DATA GM/ 1.40764665 D16 /
  ISN 0010            DATA A/ 20925672.6 D0 /
  ISN 0011            EQUIVALENCE (CJCAS,JA2),(D,COND)
  ISN 0012            P2 = X(1)*X(1) + X(2)*X(2) + X(3)*X(3)
  ISN 0013            R = DSQRT(R2)
  ISN 0014            R3 = R*R2
  ISN 0015            R4 = R2*R2
  ISN 0016            R6 = R3*R3
  ISN 0017            DO 10 I=1,3
  ISN 0018            P1(1,I) = X(I)/R
  ISN 0019            P1(2,I) = -2.D0*X(I)/R4
  ISN 0020         10 P1(3,I) = -4.D0*X(I)/R6
  ISN 0021            Z2 = X(3)*X(3)
  ISN 0022            P2(1) = -2.D0*X(1)*Z2/R4
  ISN 0023            P2(2) = -2.D0*X(2)*Z2/R4
  ISN 0024            P2(3) = 2.D0*X(3)*(R2 - Z2)/R4
  ISN 0025            P3(1,1) = (R2 - X(1)*X(1))/R3
  ISN 0026            P3(1,2) = -X(1)*X(2)/R3
  ISN 0027            P3(1,3) = -X(1)*X(3)/R3
  ISN 0028            P3(2,1) = P3(1,2)
  ISN 0029            P3(2,2) = (R2 - X(2)*X(2))/R3
  ISN 0030            P3(2,3) = -X(2)*X(3)/R3
  ISN 0031            P3(3,1) = P3(1,3)
  ISN 0032            P3(3,2) = P3(2,3)
  ISN 0033            P3(3,3) = (R2 - Z2)/R3
  ISN 0034            Z2 = Z2/R2
  ISN 0035            DA4 = D*A*A*A
  ISN 0036            H = (GM/R2)*(1.D0+(JA2/R2)*(1.D0 - 5.D0*Z2) + (DA4/R4)*
                     1     (0.42857143D0 - Z2*(6.D0 - 9.D0*Z2)))
  ISN 0037            DO 20 I=1,3
  ISN 0038            P4(I) = (GM/R2)*( (JA2/R2)*(-5.D0*P2(I)) + JA2*(1.D0 - 5.D0*Z2)
                     1          *P1(2,I) - (DA4/R4)*(Z2*(-9.D0*P2(I)) + (6.D0 - 9.D0*Z2)
                     2         *P2(I)) + DA4*(0.42857143D0 - Z2*(6.D0 - 9.D0*Z2))
                     3         *P1(3,I)) + H*R2*P1(2,I)
  ISN 0039         20 CONTINUE
  ISN 0040            GZ = -(X(3)/R)*(H + (GM/R4)*(2.D0*JA2 + (DA4/R2)
```

```
                     1     *(1.71428571400 - 4.00*Z2)))
ISN 0041              DO 30 I=1,3
ISN 0042              F(1,I) = -M*P3(1,I) - (X(1)/R)*P4(I)
ISN 0043              F(2,I) = -M*P3(2,I) - (X(2)/R)*P4(I)
ISN 0044              F(3,I) = -(X(3)/R)*(P4(I) + (GM/R4)*((DA4/R2)*(-4.00*P2(I))
                     1          + DA4*(1.71428571400 - 4.00*Z2)*P1(2,I))
                     2          + GM*(2.00*JA2 + (DA4/R2)*(1.71428571400 - 4.00*Z2))
                     3          *P1(3,I)) + (R*GZ/X(3))*P3(3,I)
ISN 0045            30 CONTINUE
ISN 0046              WSQ = W*W
ISN 0047              RETURN
ISN 0048              END

*OPTIONS IN EFFECT*       NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,

*OPTIONS IN EFFECT*       SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF

*STATISTICS*     SOURCE STATEMENTS =      47 ,PROGRAM SIZE =     1854

*STATISTICS*  NO  DIAGNOSTICS GENERATED

****** END OF COMPILATION ******                                    67K BYTES OF CORE NOT USED

*STATISTICS*  NO  DIAGNOSTICS THIS STEP
```

6.15 CON SUBROUTINE DESCRIPTION

FUNCTION: Computes constants for the Spline interpolating polynomials for the position in the state vector and for the transition matrix and its inverse

INPUT:

/VAR/

H	Step size of integration
XO	9 dimensional array containing position and velocity at the beginning of the interval
X	9 dimensional array containing position and velocity at the end of the interval
PHIO	6x6 dimensional array containing transition matrix at the beginning of the interval
PHI	6x6 dimensional array containing transition matrix at the end of the interval
PHIDO	6x6 dimensional array containing derivatives of PHIO
PHID	6x6 array containing derivatives of PHI
PHINO	6x6 array containing the inverse of PHIO
PHINDO	6x6 array containing derivatives of PHINO
PHIN	6x6 array containing the derivatives of PHIN
PHIND	6x6 array containing the derivatives of PHIN
ID	FLAG TO COMPUTE COEFFICIENTS FOR SPLINES FOR TRANSITION INVERSE

OUTPUT:

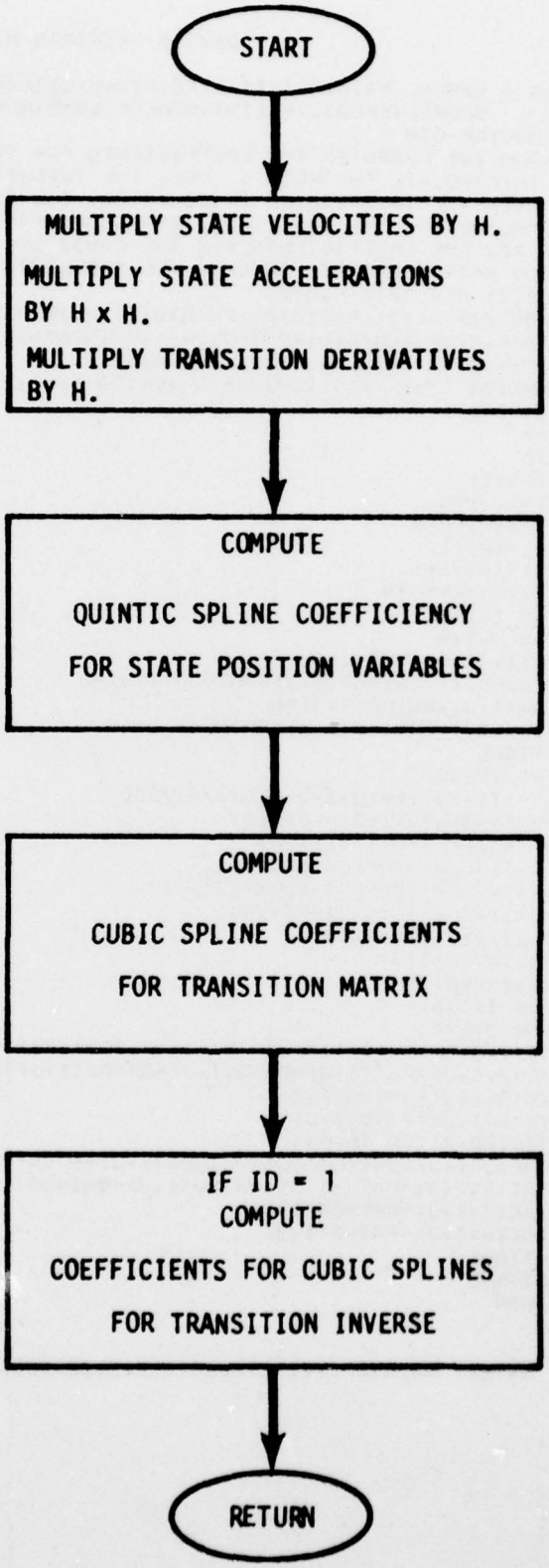
/VAR/

XC	6x3 array containing coefficients for the quintic Splines for position. The first index indicates the polynomial coefficient and the second indicates the variable
PHIC	4x6x6 array containing coefficients for the cubic Splines for the transition matrix. The first index indicates the coefficient and the second and third indices indicates the matrix element.

PHINC 4x6x6 array containing coefficients for the cubic Splines for the inverse of the Transition matrix. The indcies are the same for RHIC

SUBROUTINES USED: None

CON FLOWCHART



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LEVEL 21.7 ( JAN 73 )                    OS/360  FORTRAN H                    DATE

          COMPILER OPTIONS - NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
                             SOURCE,EBCDIC,NOLIST,NODECK,LOAD,NOMAP,NOEDIT,ID,NOXREF
   ISN 0002            SUBROUTINE CON
                     C THIS SUBROUTINE COMPUTES THE COEFFICIENTS FOR THE SPLINE POLYNOMIALS
                     C USED TO INTERPOLATE THE POS. & VEL. THE TRANSITION MATRIX AND ITS
                     C INVERSE OVER THE INTERVAL OF INTEGRATION. THE COEFFICIENTS OF THE
                     C QUINTIC SPLINE FOR THE THREE POSITION COORDINANTS ARE STORED IN THE
                     C VARIABLE XC. THE COEFFICIENTS FOR THE CUBIC SPLINES FOR THE
                     C TRANSITION MATRIX AND ITS INVERSE ARE STORED IN PHIC AND PHINC.
   ISN 0003            IMPLICIT REAL*8(A-H,O-Z)
   ISN 0004            COMMON /VAR/X(9),PHI(6,6),PHID(6,6),PHIN(6,6),PHIND(6,6),
                      1 PHIO(6,6),PHIDO(6,6),PHINO(6,6),PHINDO(6,6),XO(9),XC(6,3),
                      2 PHIC(4,6,6),PHINC(4,6,6),TS,T,H,ID
   ISN 0005            DIMENSION Y(9),YO(9),QHID(6,6),QHIDO(6,6),QHIND(6,6),QHINDO(6,6)
   ISN 0006            DO 35 I=1,3
   ISN 0007            J=I+3
   ISN 0008            K=I+6
   ISN 0009            Y(I)=X(I)
   ISN 0010            Y(J)=X(J)*H
   ISN 0011            Y(K)=X(K)*H*H
   ISN 0012            YO(I)=XO(I)
   ISN 0013            YO(J)=XO(J)*H
   ISN 0014         35 YO(K)=XO(K)*H*H
   ISN 0015            DO 40 I=1,6
   ISN 0016            DO 40 J=1,6
   ISN 0017            QHID(I,J)=PHID(I,J)*H
   ISN 0018            IF(ID.EQ.1) QHINDO(I,J)=PHINDO(I,J)*H
   ISN 0020            QHIDO(I,J)=PHIDO(I,J)*H
   ISN 0021            IF(ID.EQ.1) QHIND(I,J)=PHIND(I,J)*H
   ISN 0023         40 CONTINUE
   ISN 0024            DO 45 I=1,3
   ISN 0025            EL1=Y(I)-YO(I)-YO(I+3)-YO(I+6)/2D0
   ISN 0026            EL2=Y(I+3)-YO(I+3)-YO(I+6)
   ISN 0027            EL3=(Y(I+6)-YO(I+6))/2D0
   ISN 0028            XC(1,I)=EL3-3D0*EL2+6D0*EL1
   ISN 0029            XC(2,I)=EL2-3D0*EL1-2D0*XC(1,I)
   ISN 0030            XC(3,I)=EL1-XC(1,I)-XC(2,I)
   ISN 0031            XC(4,I)=YO(I+6)/2D0
   ISN 0032            XC(5,I)=YO(I+3)
   ISN 0033         45 XC(6,I)=YO(I)
   ISN 0034            DO 50 I=1,6
   ISN 0035            DO 50 J=1,6
   ISN 0036            PHIC(1,I,J)=QHID(I,J)+QHIDO(I,J)-2D0*(PHI(I,J)-PHIO(I,J))
   ISN 0037            PHIC(2,I,J)=PHI(I,J)-PHIO(I,J)-QHIDO(I,J)-PHIC(1,I,J)
   ISN 0038            PHIC(3,I,J)=QHIDO(I,J)
   ISN 0039            PHIC(4,I,J)=PHIO(I,J)
   ISN 0040            IF(ID.EQ.0) GO TO 49
   ISN 0042            PHINC(1,I,J)=QHIND(I,J)+QHINDO(I,J)-2D0*(PHIN(I,J)-PHINO(I,J))
   ISN 0043            PHINC(2,I,J)=PHIN(I,J)-PHINO(I,J)-QHINDO(I,J)-PHINC(1,I,J)
   ISN 0044            PHINC(3,I,J)=QHINDO(I,J)
   ISN 0045            PHINC(4,I,J)=PHINO(I,J)
   ISN 0046         49 CONTINUE
   ISN 0047         50 CONTINUE
   ISN 0048            RETURN
   ISN 0049            END

*OPTIONS IN EFFECT*      NAME=  MAIN,OPT=01,LINECNT=60,SIZE=0000K,
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REFERENCES

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